Parameterized Algorithms for Scalable Interprocedural Data-flow Analysis

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Agenda

1. Motivation
2. The IFDS framework
3. Sparsity parameters
4. Solving IFDS problems
5. Experimental results

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Maiden flight of Ariane 5

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Maiden flight of Ariane 5

40 seconds later...

Maiden flight of Ariane 5

The rocket self destruct due to a software error: an unsafe conversion from 64-bit float to a 16-bit integer was not caught and led to uncontrollable behavior.

This error cost US$370 million.

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"I call it my billion-dollar mistake. It was the invention of the null reference in 1965. [...] This has led to innumerable errors, vulnerabilities, and system crashes, which have probably caused a billion dollars of pain and damage in the last forty years."

- Tony Hoare

Static program analysis

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Static program analysis: the science of automatically finding bugs in programs without running them.

Uses of static program analysis

Static program analysis attempts to answer question like:

- Does the program use a variable `x` before it is initialized?
- Can the program have a null-pointer dereferencing?
- If expression `e` is inside a loop, does `e`'s value depend on the loop iteration?

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Applications:

- Optimizing compilers.
- IDEs.
- Verification of safety-critical systems.
  - In 2003, Astrée was used to verify the flight control software of Airbus A340.

Overview

We will consider the IFDS framework, which captures a large class of useful static analyses such as:

- possibly-uninitialized variables,
- null-pointer,
- reaching definitions,
- available expressions,
- live variables, and
- dead-code elimination.

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Setting:

- *Large scale.* We have a large codebase (e.g., in Google/Meta) on which we want to perform some IFDS analysis.
- *On-demand.* We receive a large stream of queries (e.g., from developers) inquiring about the analysis result between two particular statements in the codebase.

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Standard IFDS algorithms. Authors of IFDS (POPL'95 [1], FSE'95 [2]) gave algorithms to achieve this, but they do not scale to large codebases with over 10^5 LoC.

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- Pro: fast preprocessing and query time.
- Con: they solve a restricted case of the problem.

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This work: exploit low treedepth of call graphs to solve the general case.

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For a program of n lines:

Approach	General?	Preprocessing	Query
Reps et. al. (POPL'95)	✓	$O(n)$	
Horwitz et. al. (FSE'95)	✓	$O(n)$	
Chatterjee et. al. (ESOP'20)	✗	$O(n)$	$O(1)$
Our result	✓	$O(n)$	$O(1)$

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Abstractions for programs

We'll need 3 abstractions to formalize the structure of a program:

1. Control-flow graphs.
2. Supergraphs.
3. Call graphs.

Control flow graphs (CFGs)

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- G_f has a *start* vertex s_f and *exit* vertex e_f.

```
1 int add(int a, int b) {
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5         b = b - 1;
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7     return sum;
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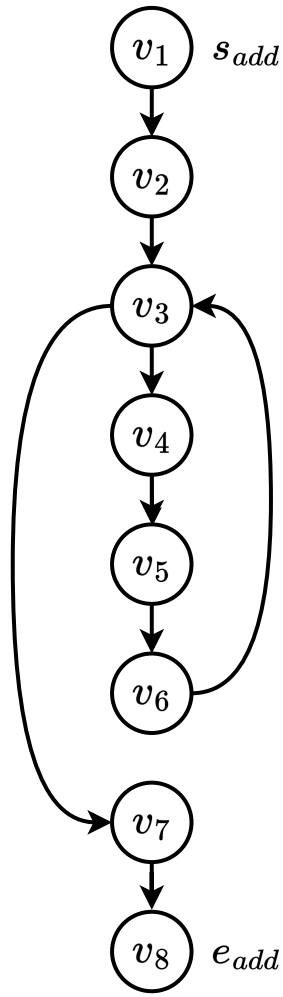

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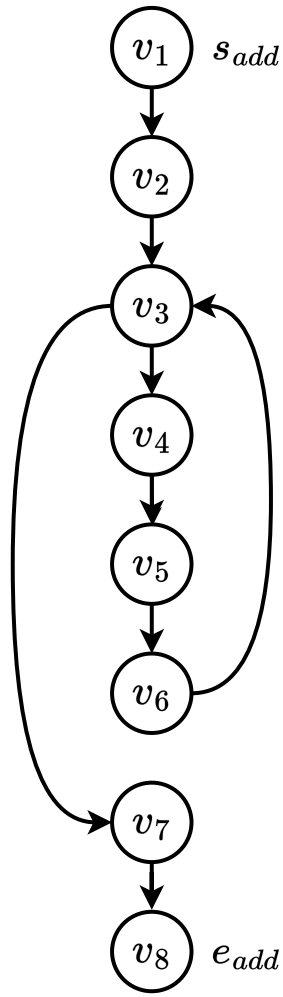

Observe:

- A path in $G_f \equiv$ an execution of P.
- The paths in G_f completely characterize f's behavior at runtime.

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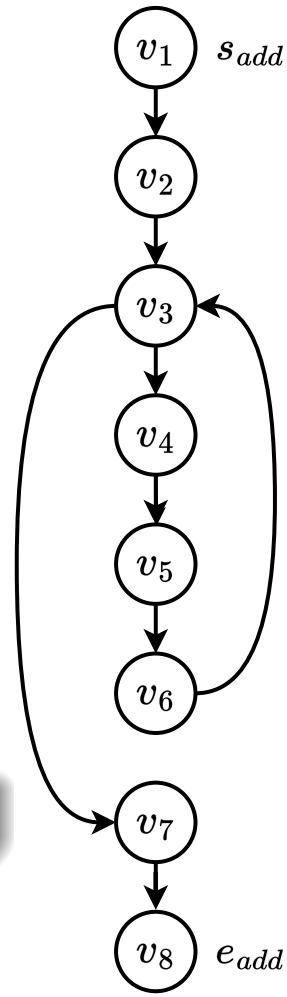

Analyzing $P \equiv$ compute the meet-over-all-paths (MOP).

Supergraphs

Program consisting of functions $f_1, \ldots, f_k$ is formalized by a supergraph G

$$G \equiv \text{ CFGs } G_{f_1}, \ldots, G_{f_k} + \textit{interprocedural edges}$$

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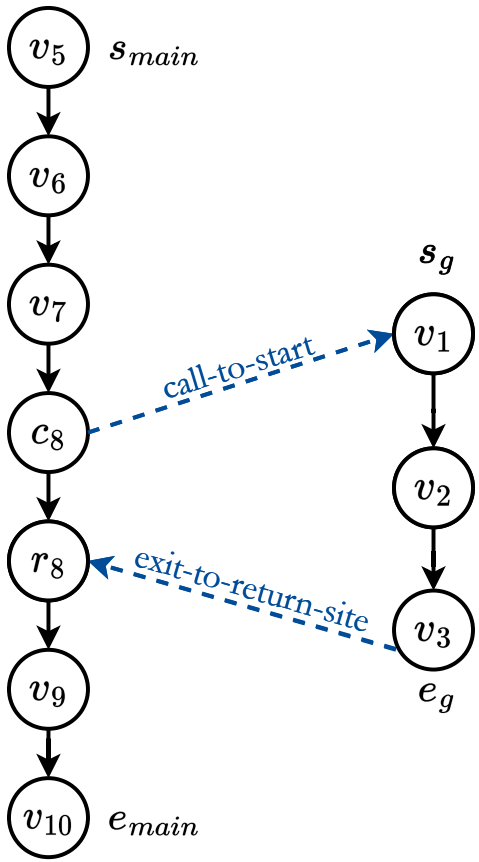

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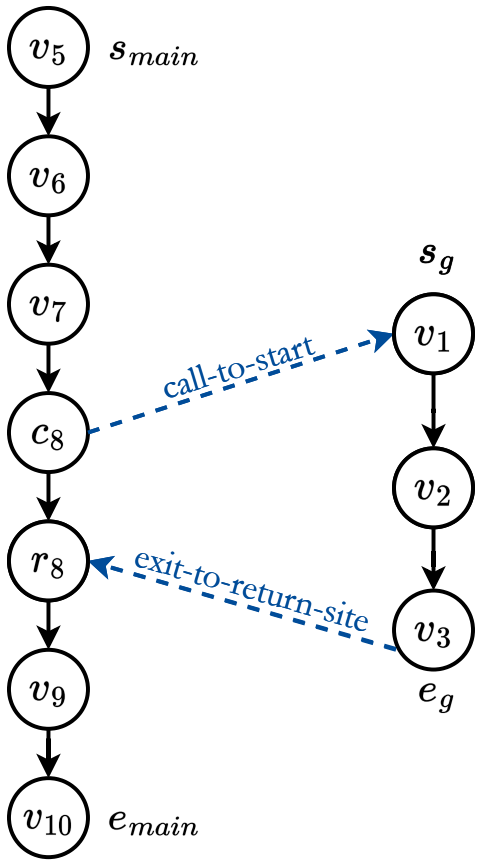

Question: a path in $G \equiv$ an execution of P?

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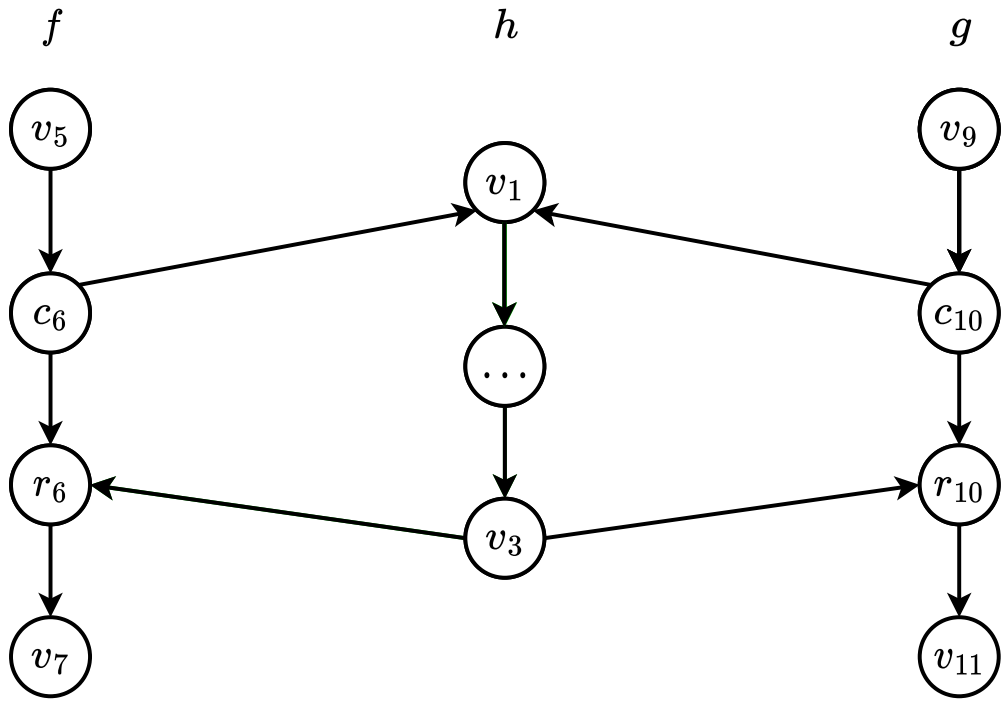

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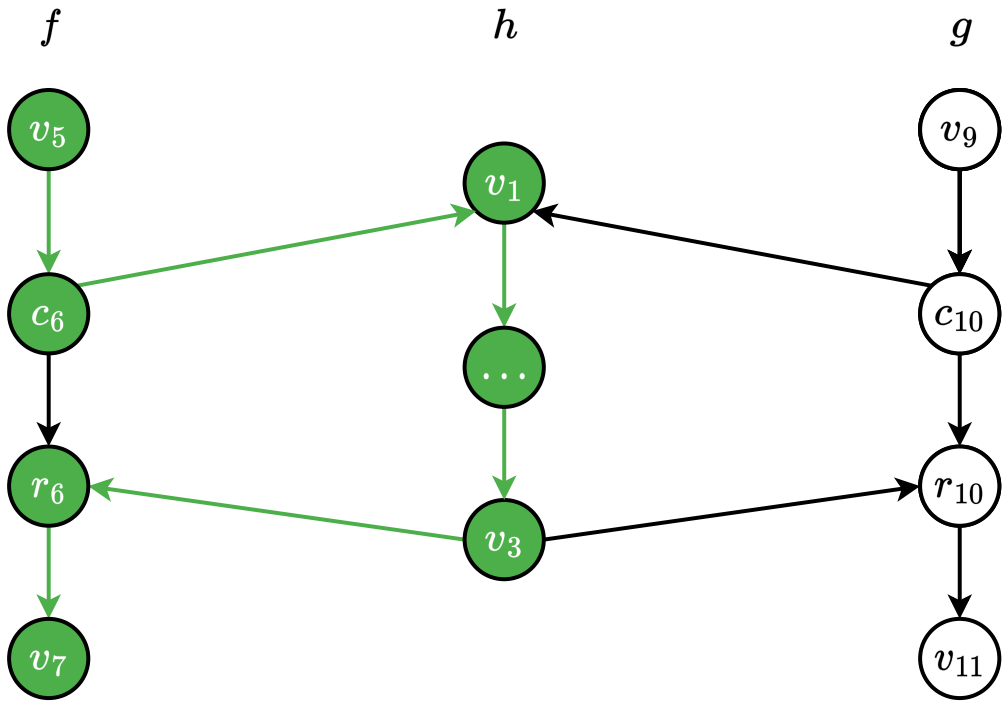

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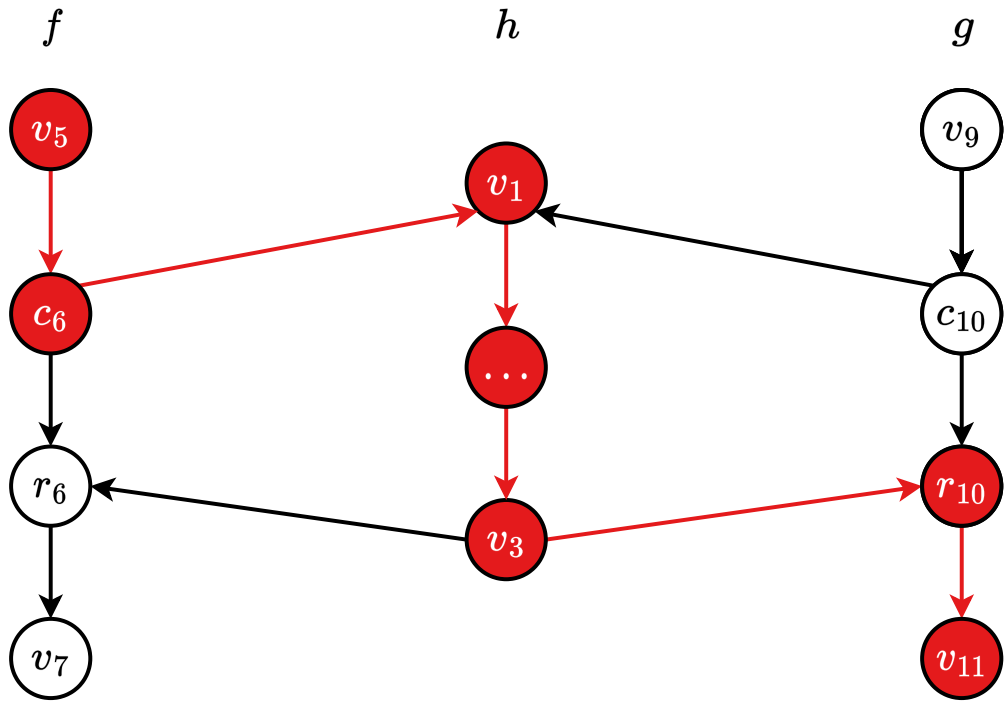

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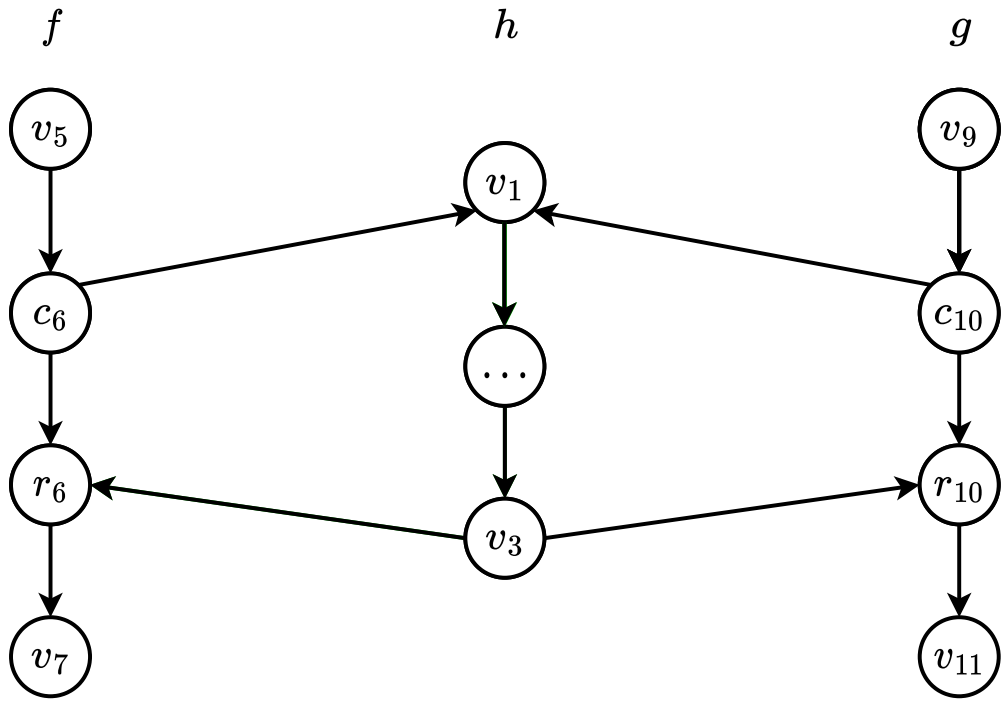

- An interprocedurally valid path (IVP) is a path where returns are to the correct matching calls.
- An IVP in $G \equiv$ an execution of P.

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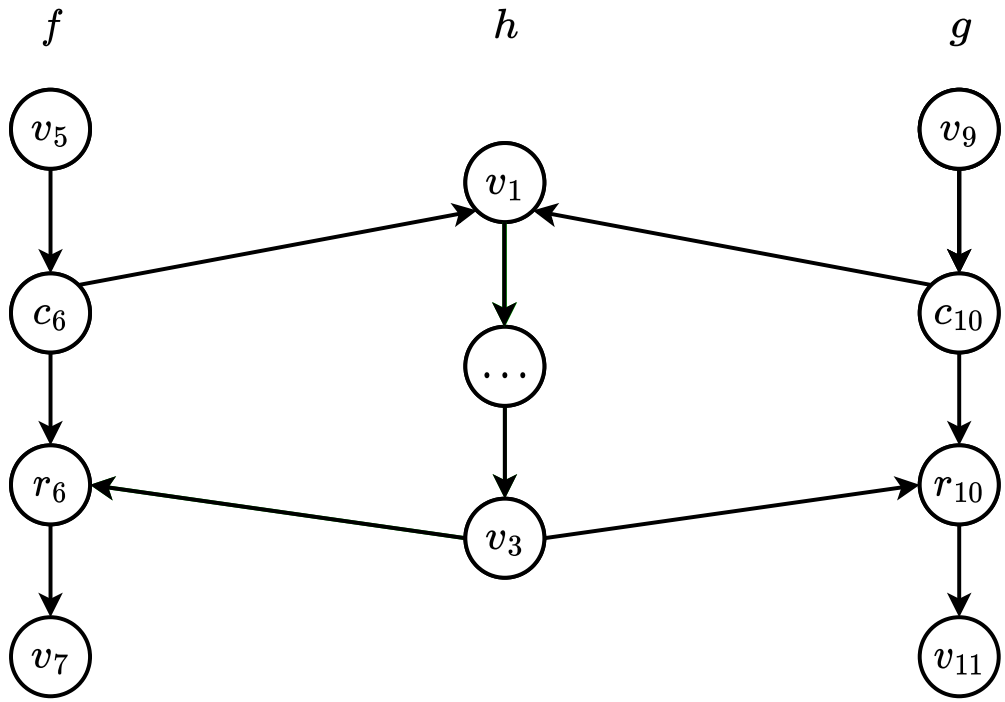

Analyzing $P \equiv$ compute the meet-over-all-valid-paths (MIVP).

Call graphs

A *Call graph* $C = (F, E_C)$ has:

- Vertices are functions of the program.
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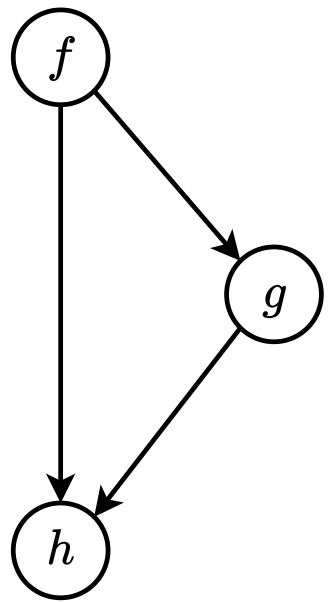

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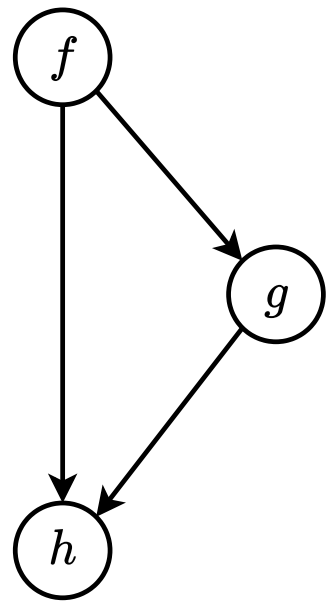

- The call graph can be inferred from the supergraph.
- Describes program's behavior at the function level.

IFDS problems

In an IFDS problem, the input is:

- Supergraph $G = (V, E)$.
- Finite set of *data-facts* D.
- Each edge $e = (I, I') \in E$ has a flow function $M_e : 2^D \to 2^D$.
- *Meet* operator $\sqcap \in \{\cup, \cap\}$.
- M_e distributes over $\sqcap$, i.e., $M_e(D_1 \sqcap D_2) = M_e(D_1) \sqcap M_e(D_2)$.

To solve the IFDS problem: compute at each program point (vertex) which data-facts hold.

IFDS problems: example

Example: null-pointer analysis.

- $D =$ set of variables in the program.
- A solution: for every $l \in V$, compute $S_l \subseteq D$, the set of variable that may be null after l if we start execution from `main`.

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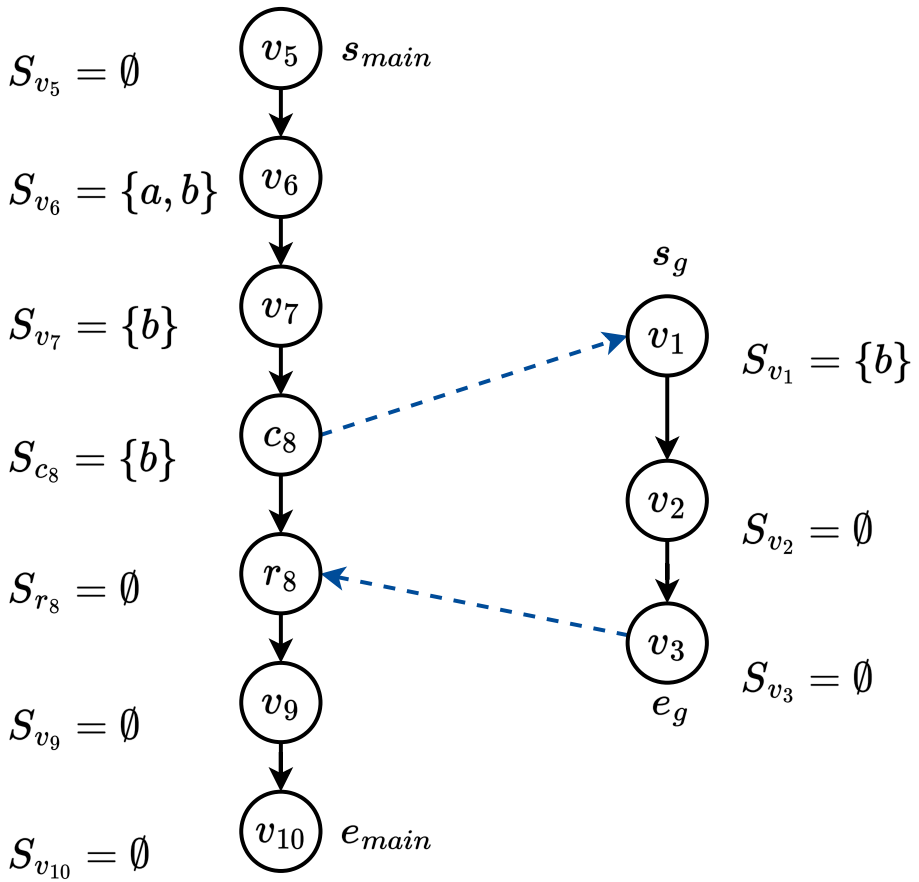


IFDS problems: formalizing a solution

More formally:

- For a path $\pi = e_1 \cdot e_2 \cdots e_k$ in G, define:

$$M_\pi = M_{e_k} \circ M_{e_k} \circ \cdots \circ M_{e_1}.$$

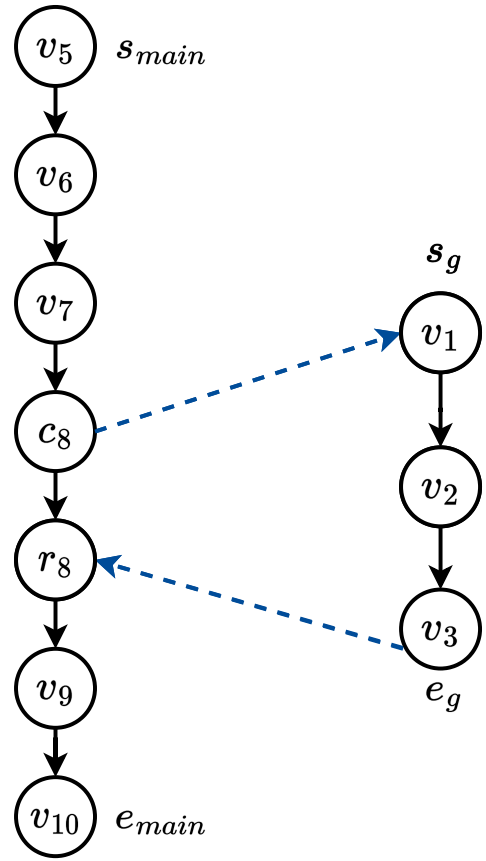
- For $u_1, u_2 \in V$, define:

$$\texttt{IVP}(u_1,u_2) = \{P \mid P \text{ is an IVP from } u_1 \text{ to } u_2 \text{ in } G\}.$$

- For $u_1, u_2 \in V$ and $D_1 \subseteq D$, we want to compute:

$$\texttt{MIVP}(u_1, D_1, u_2) := \bigsqcap_{\pi \in \texttt{IVP}(u_1,u_2)} M_\pi(D_1)$$

- We'll assume wlog that $\sqcap = \cup$.



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Input: $\langle G = (V, E), D, \{M_e\}_{e \in E} \rangle$ queries of the form $\langle u_1, D_1, u_2 \rangle$.

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In the remainder of this talk, we will give a series of observations, each of which give us a simpler problem to solve.

Where we are

Computing
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Where we are

But first, let's introduce sparsity parameters..

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Treewidth

- *Treewidth* measures "tree-likeness" of a graph.
- Graphs of small treewidth are tree-like.
- Graph problems are typically easier on trees.
- Similarly, graph problems are also typically easier on graphs of low treewidth.

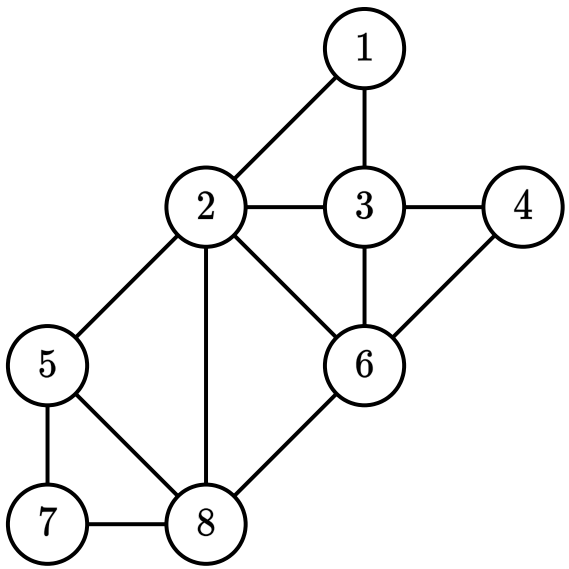
Treewidth

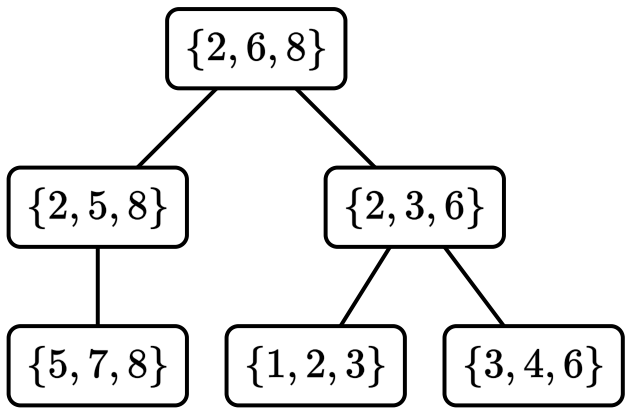
Tree Decompositions (TDs)

Given $G = (V, E)$, a *tree decomposition* of G is a tree $T = (\mathfrak{B}, E_T)$:

1. Every node $b \in \mathfrak{B}$ of the tree T has a corresponding *bag* $V_b \subseteq V$.
2. $\bigcup_{b \in \mathfrak{B}} V_b = V$.
3. $\forall u, v \in V, \{u, v\} \in E \implies \exists b \in \mathfrak{B} \ \ \{u, v\} \subseteq V_b$.
4. $\forall v \in V, \{b \in \mathfrak{B} | v \in V_b\}$ forms a connected subtree of T.

The width of T is $\max_{b \in \mathfrak{B}} |V_b| - 1$.





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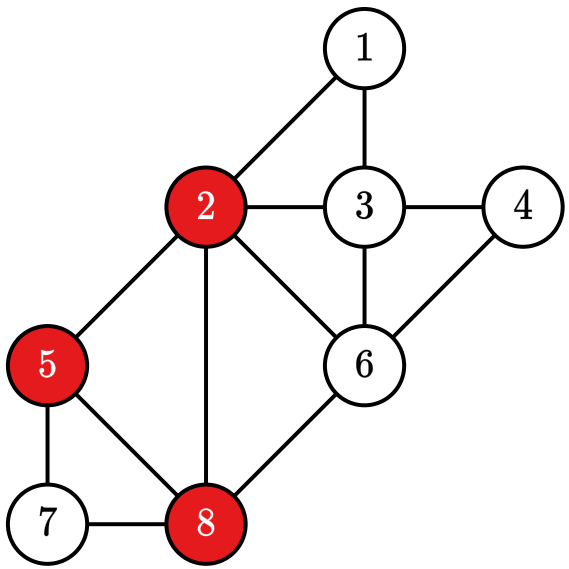

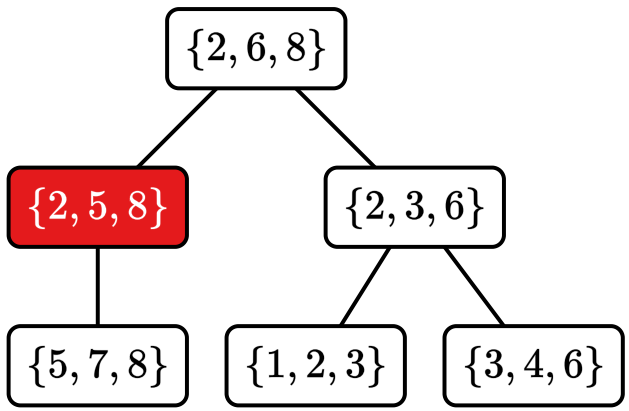

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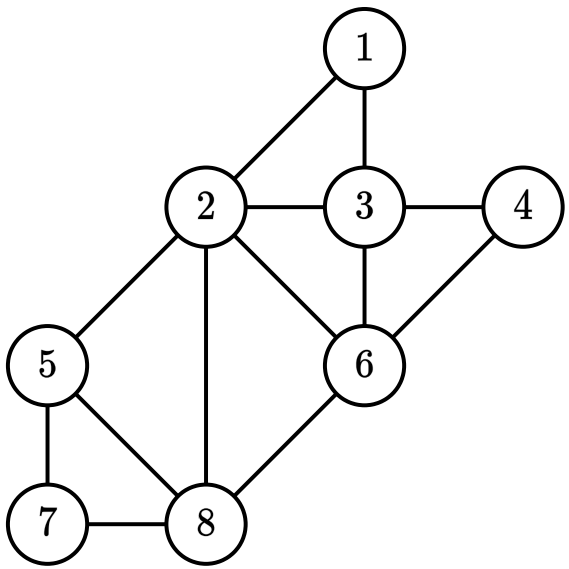

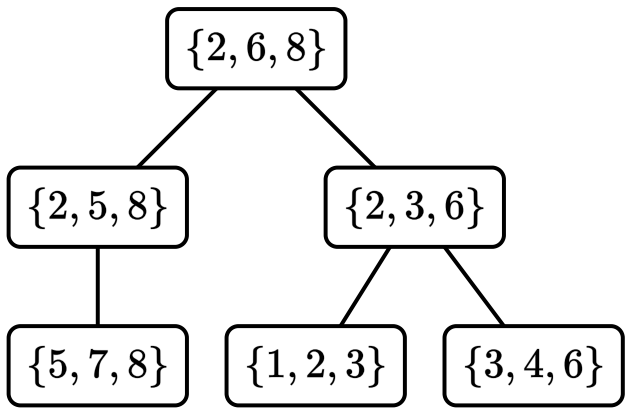

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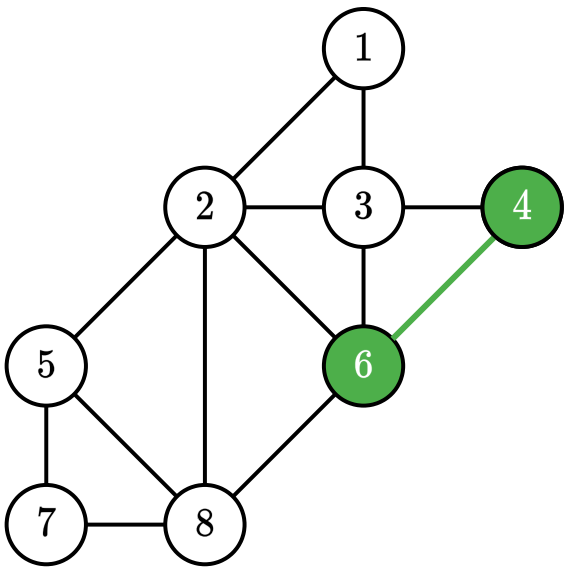

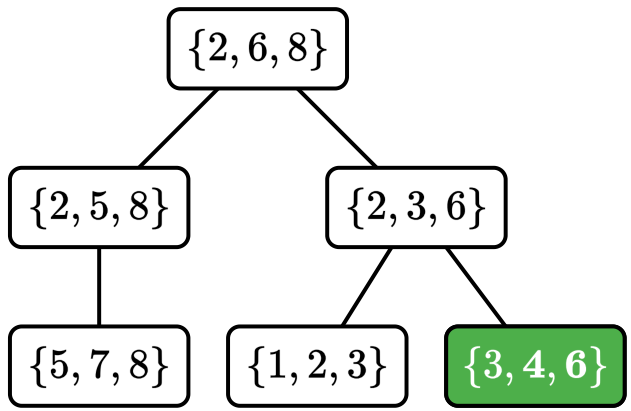

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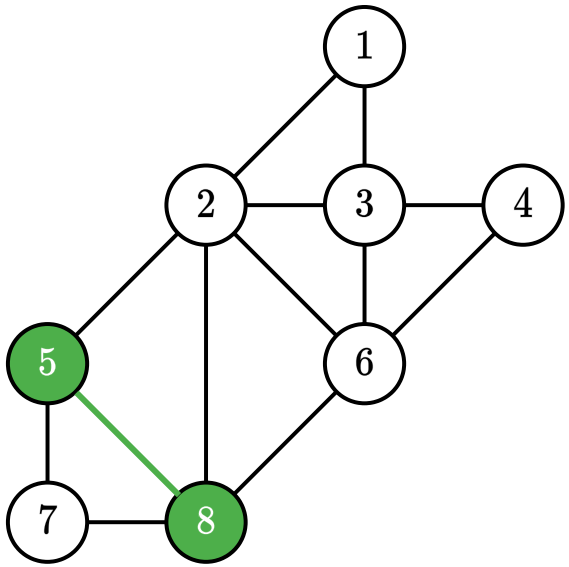

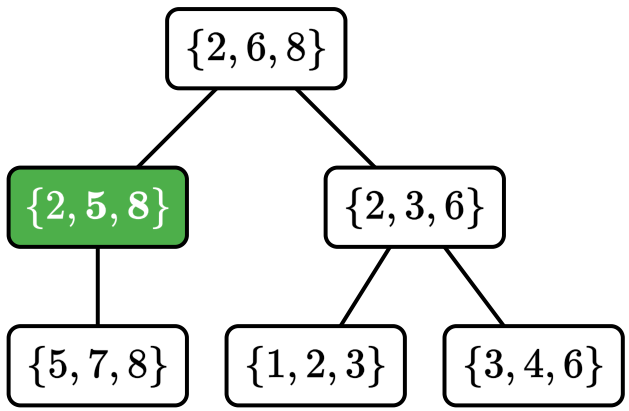

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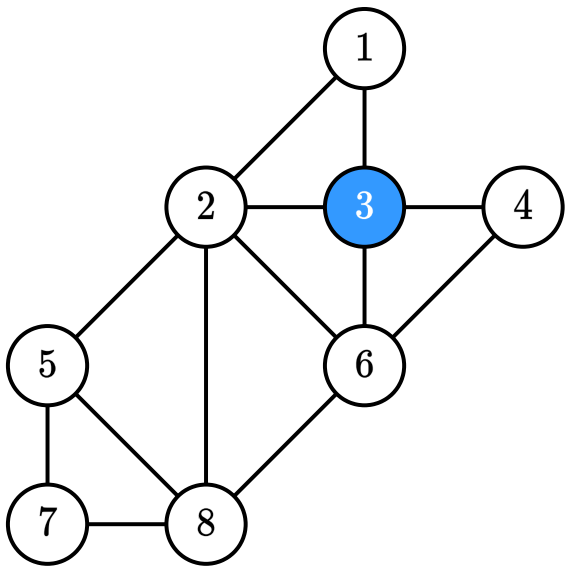

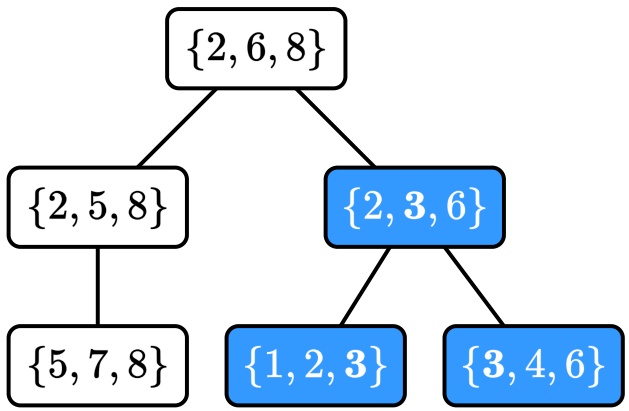

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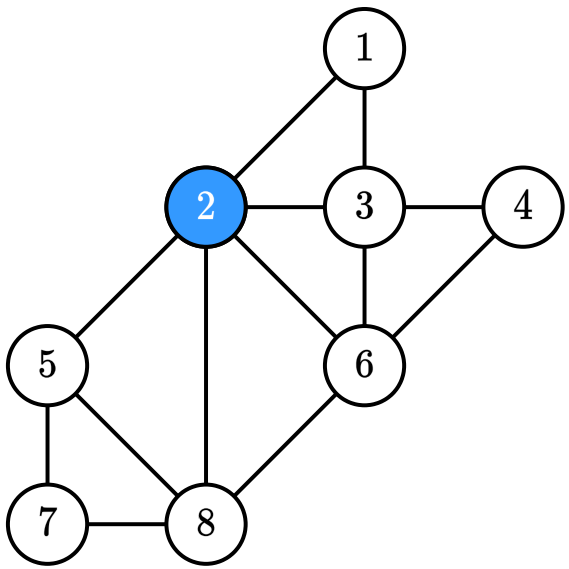

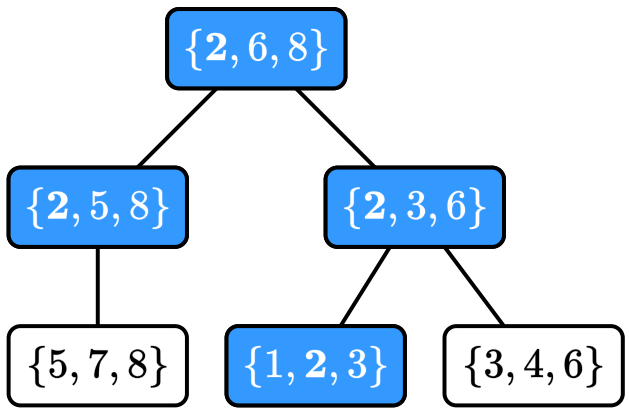

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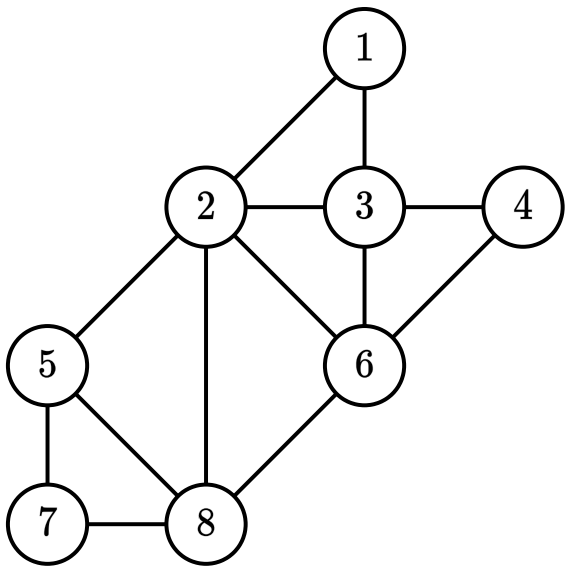

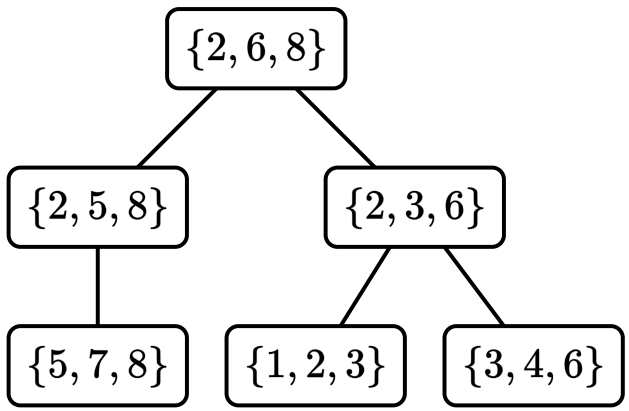

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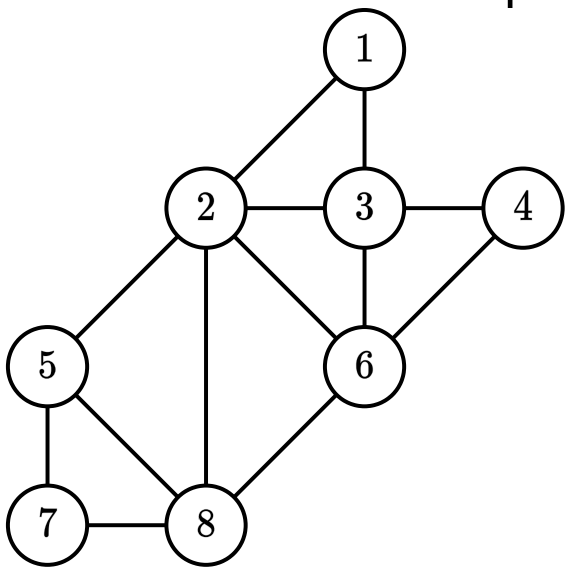
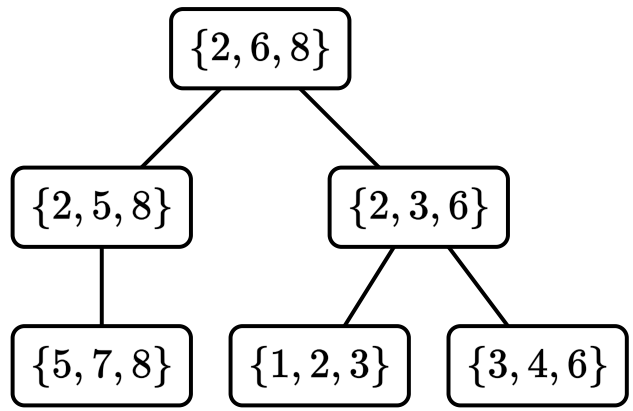
The *treewidth* of G is the smallest width among all TDs over G.

This TD has width 2 and is optimal $\implies$ the graph has treewidth 2.





Cut property of TDs

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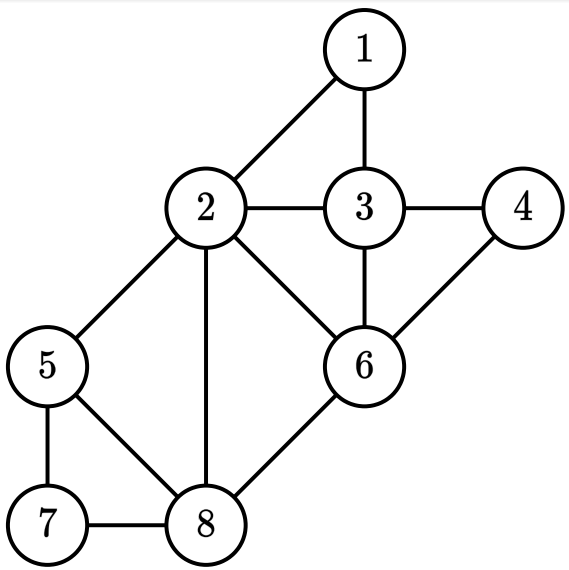

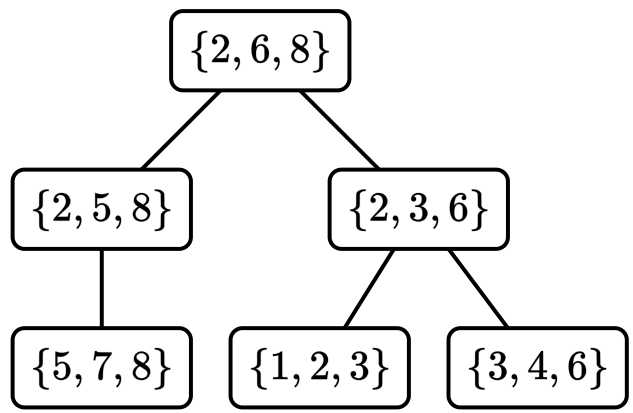

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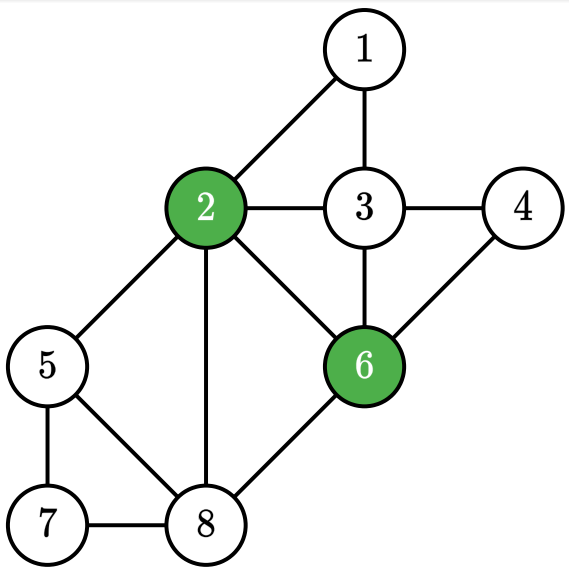

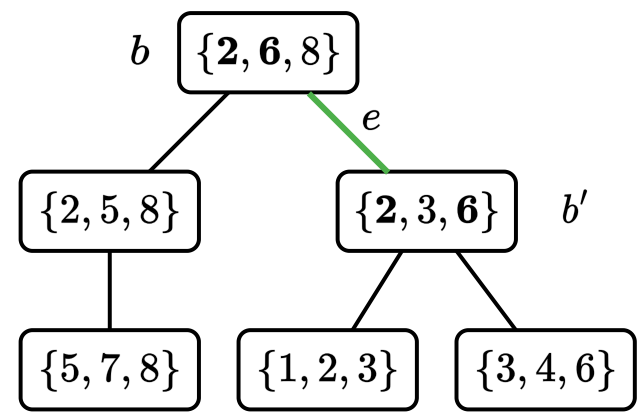

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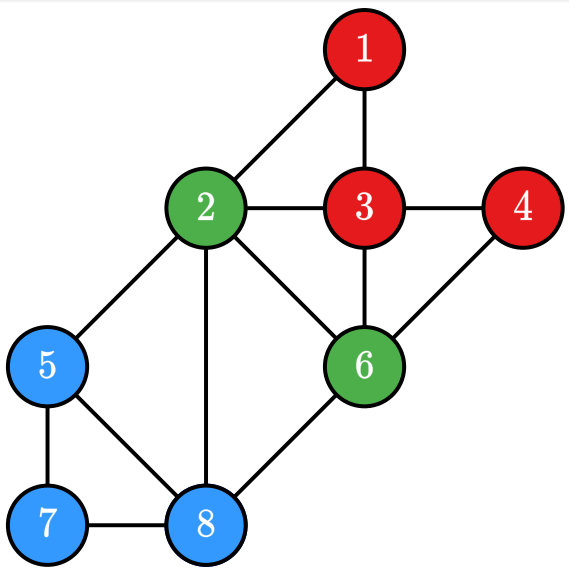

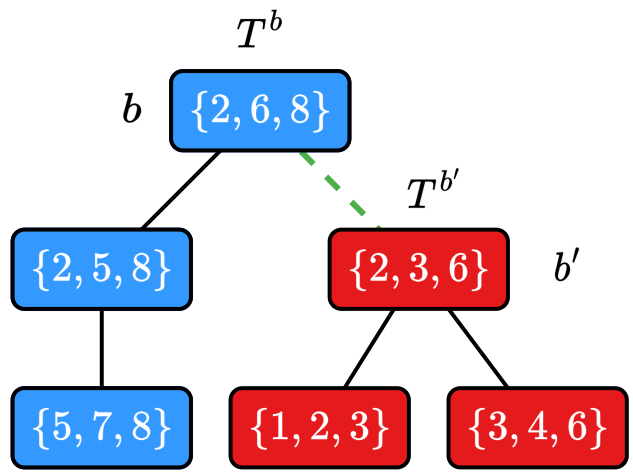

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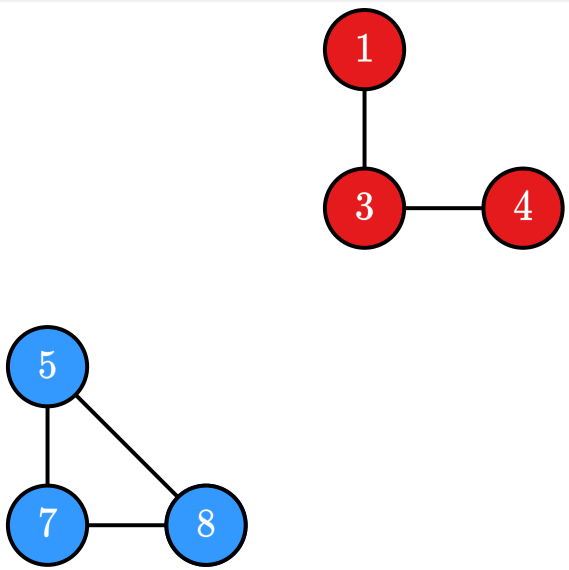
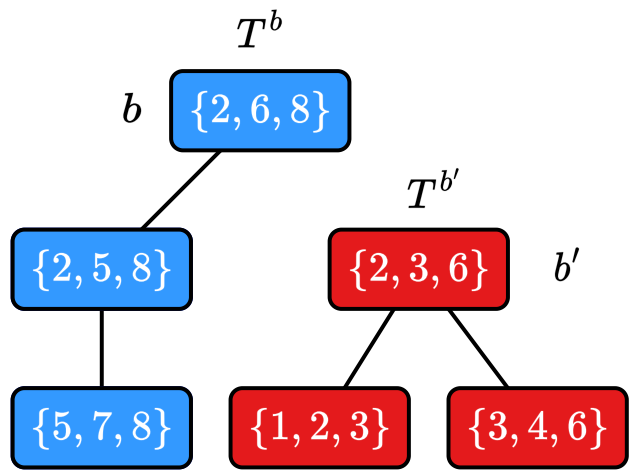




Treedepth

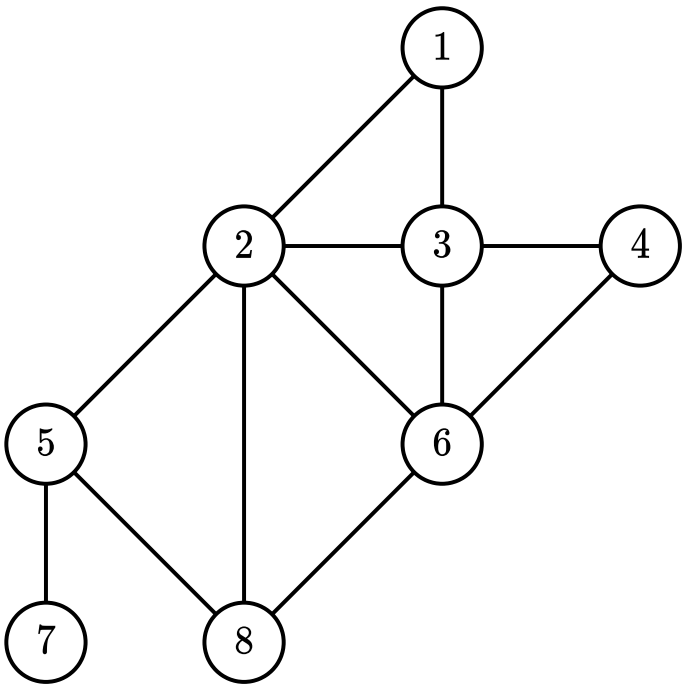
- *Treedepth* measures how much a graph resembles a *shallow tree*.
- Graphs of small treedepth have simpler structure.

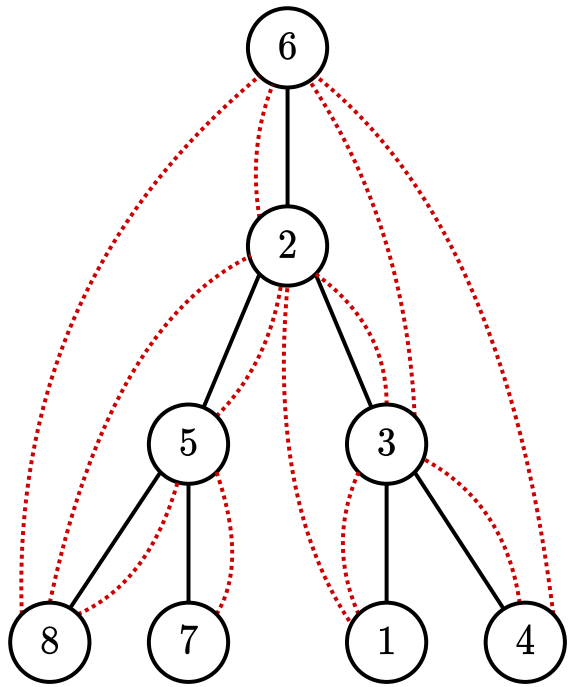
Treedepth

Partial Order Trees (POTs)

Given a graph $G = (V, E)$, a *partial order tree* over G is a *rooted* tree $T = (V, E_T)$ where

$(u, v) \in E \implies u$ and v are in an ancestor-descendant relationship in T.





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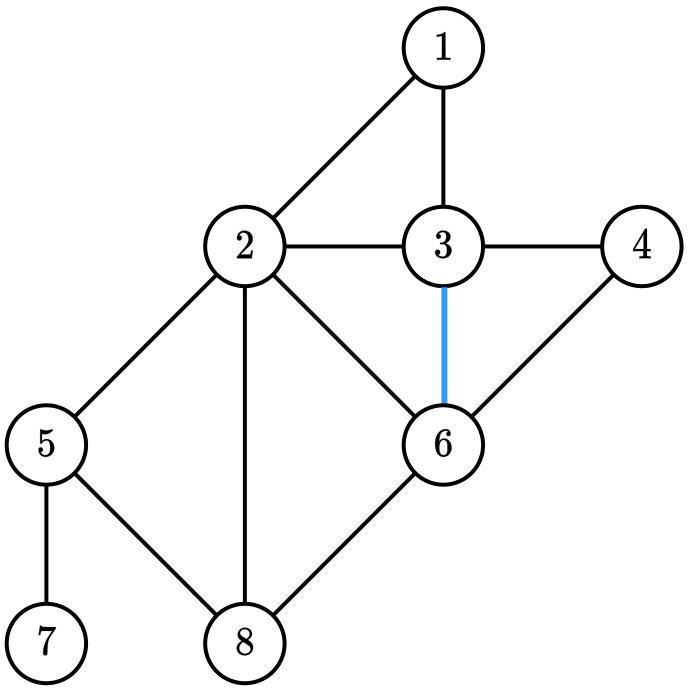

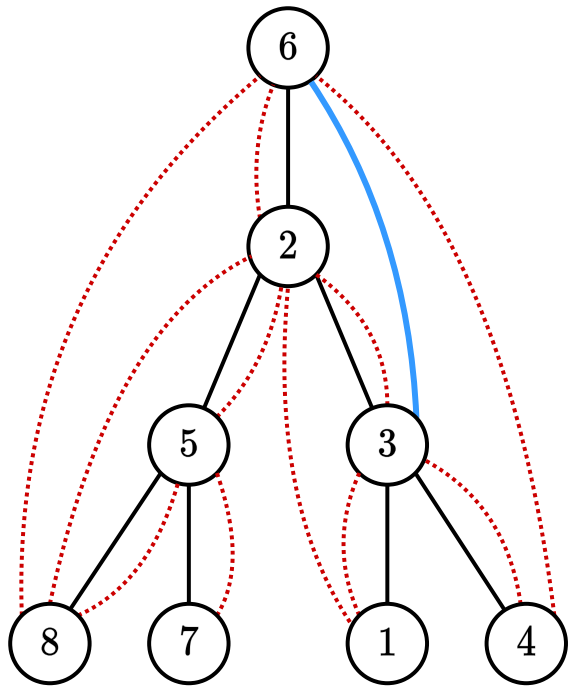

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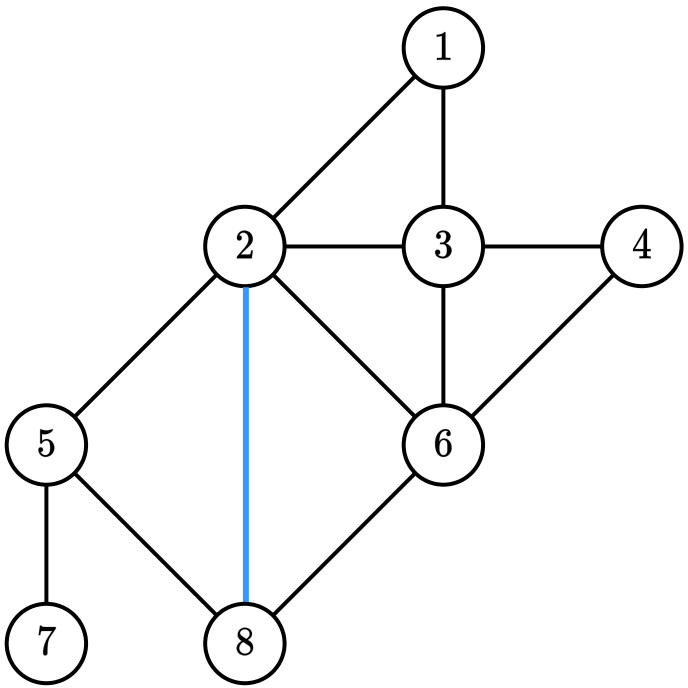

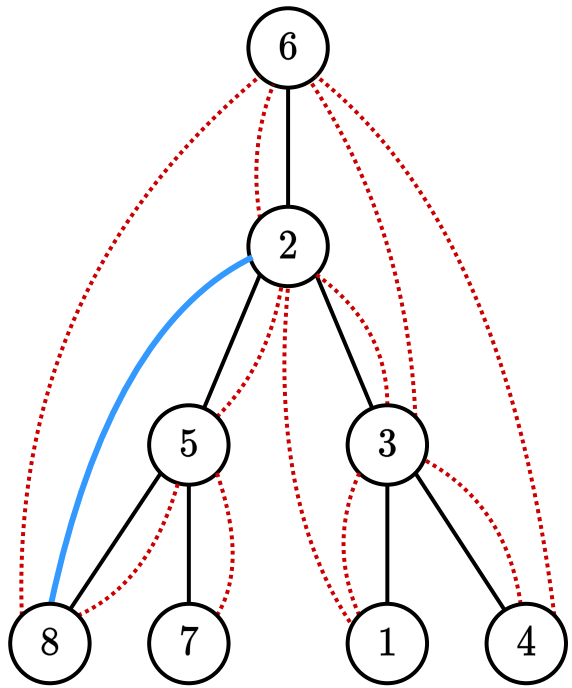

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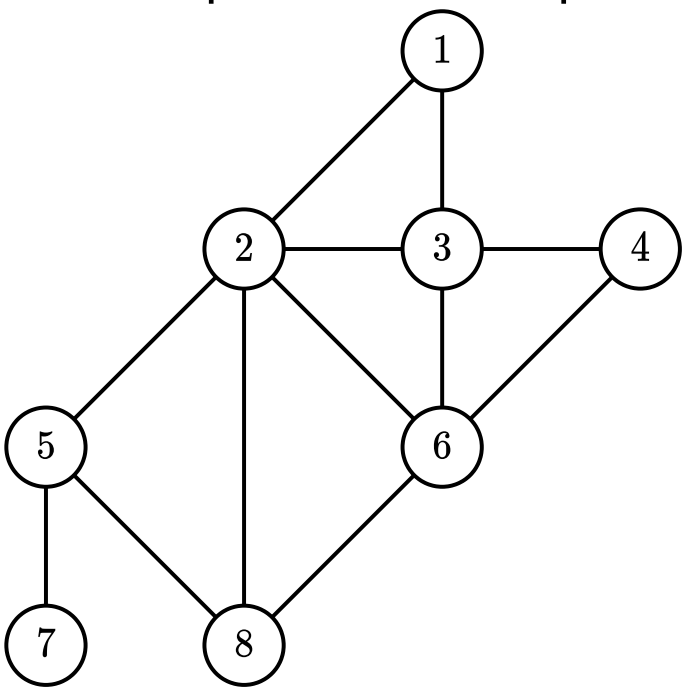

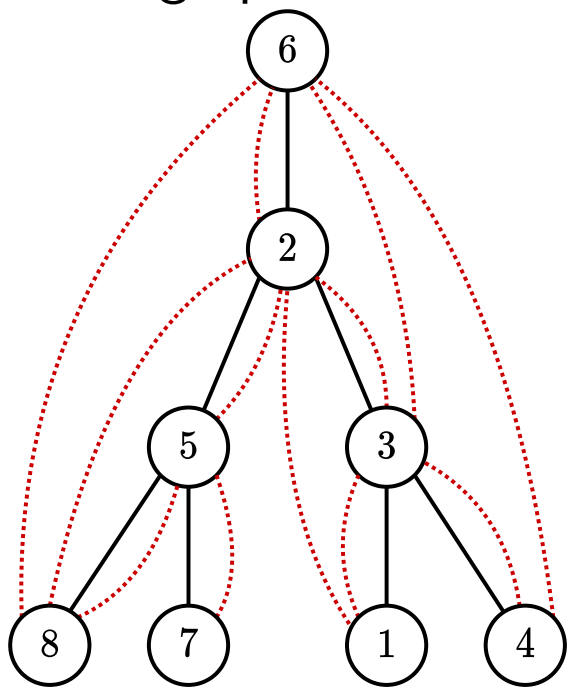

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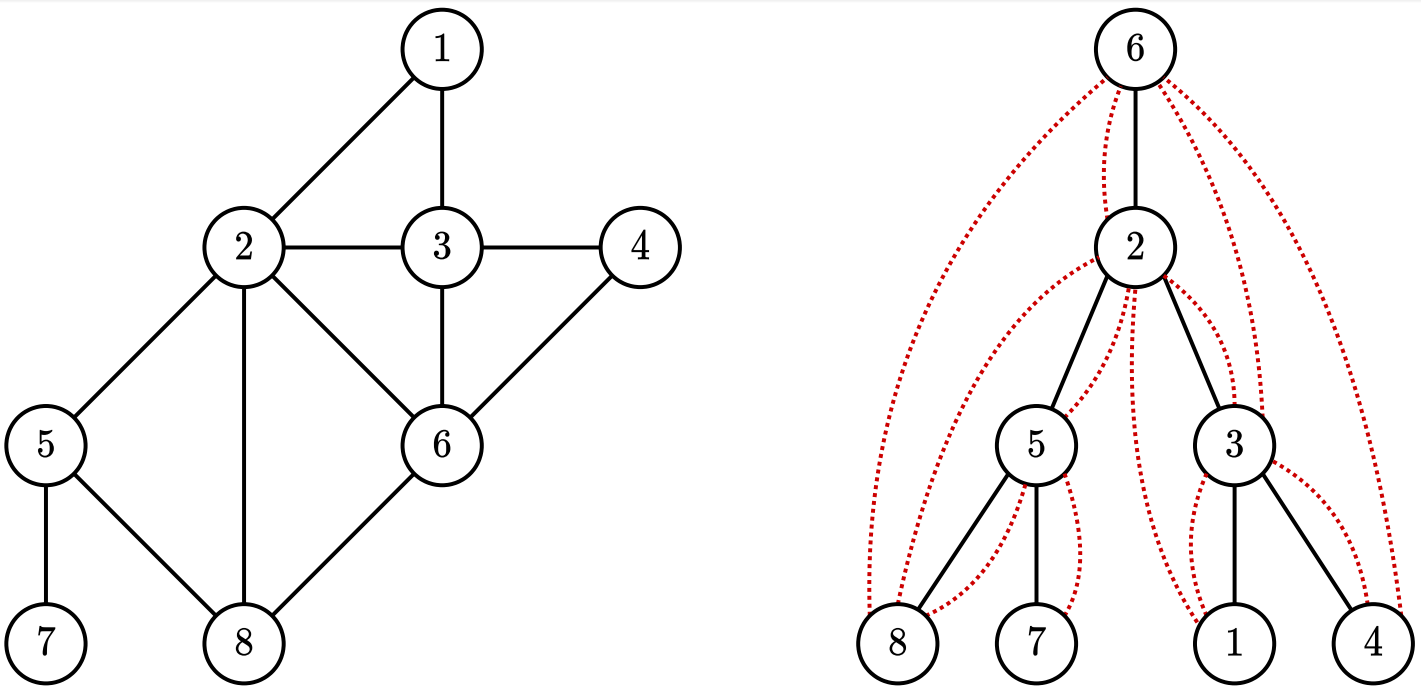

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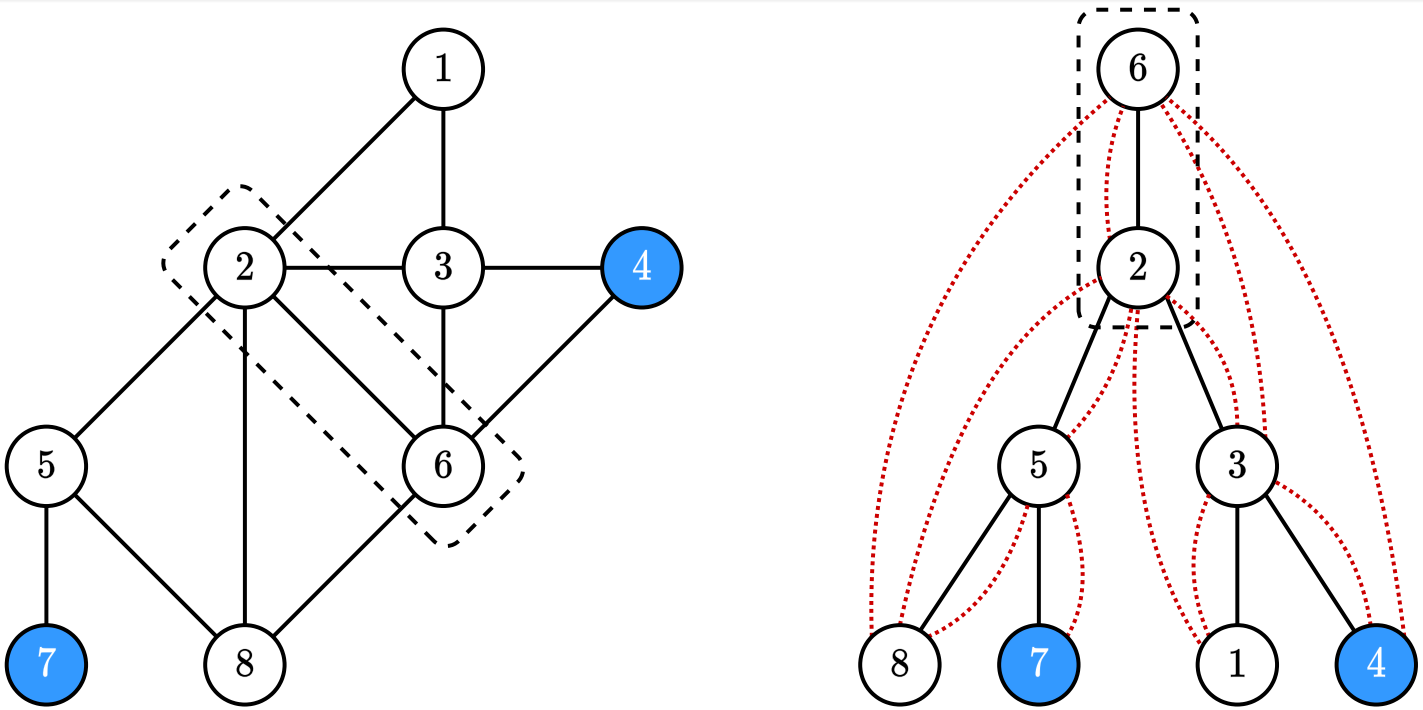

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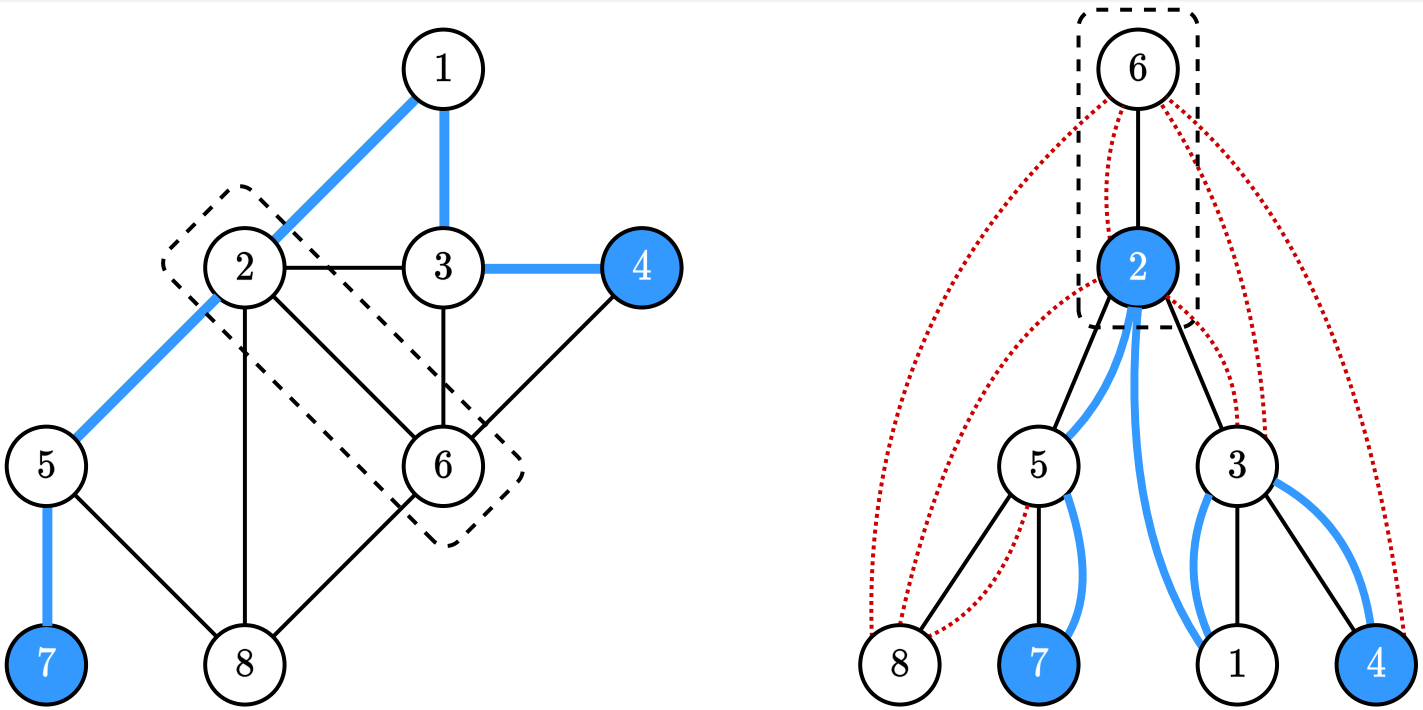

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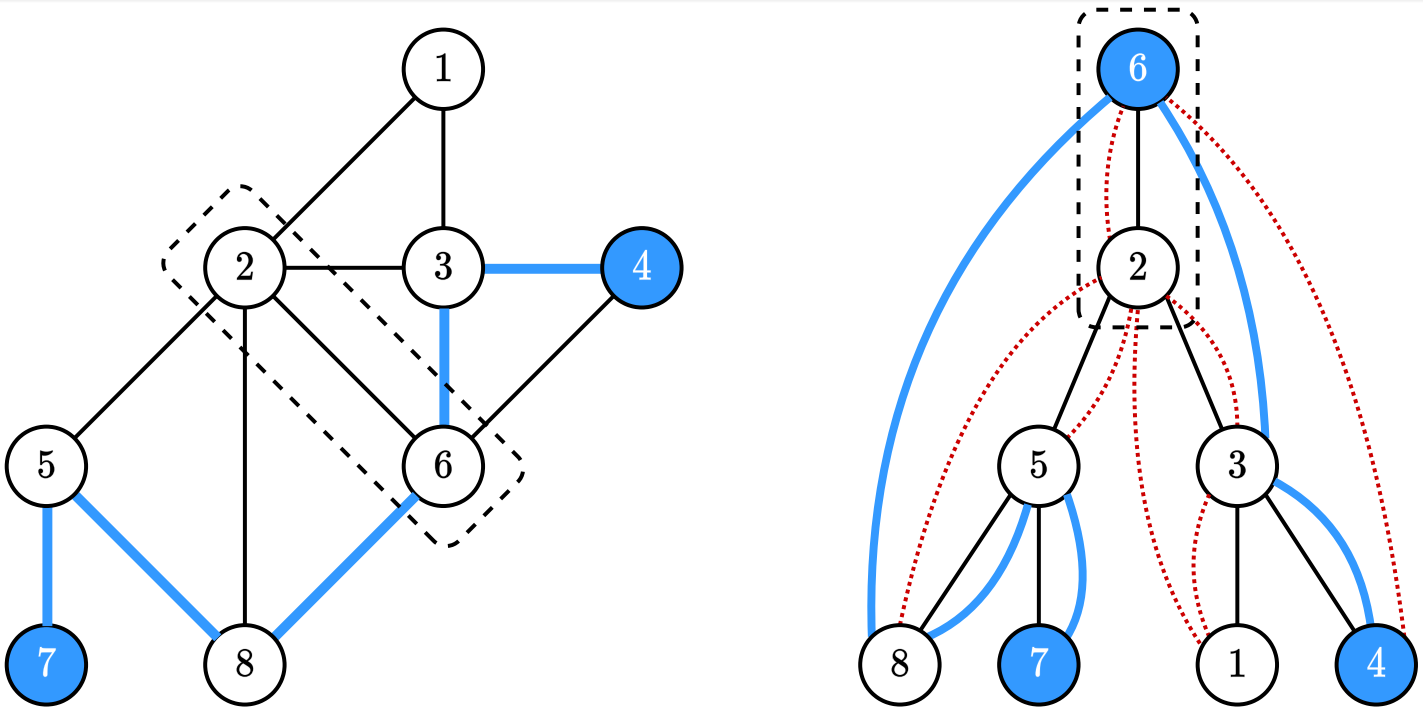

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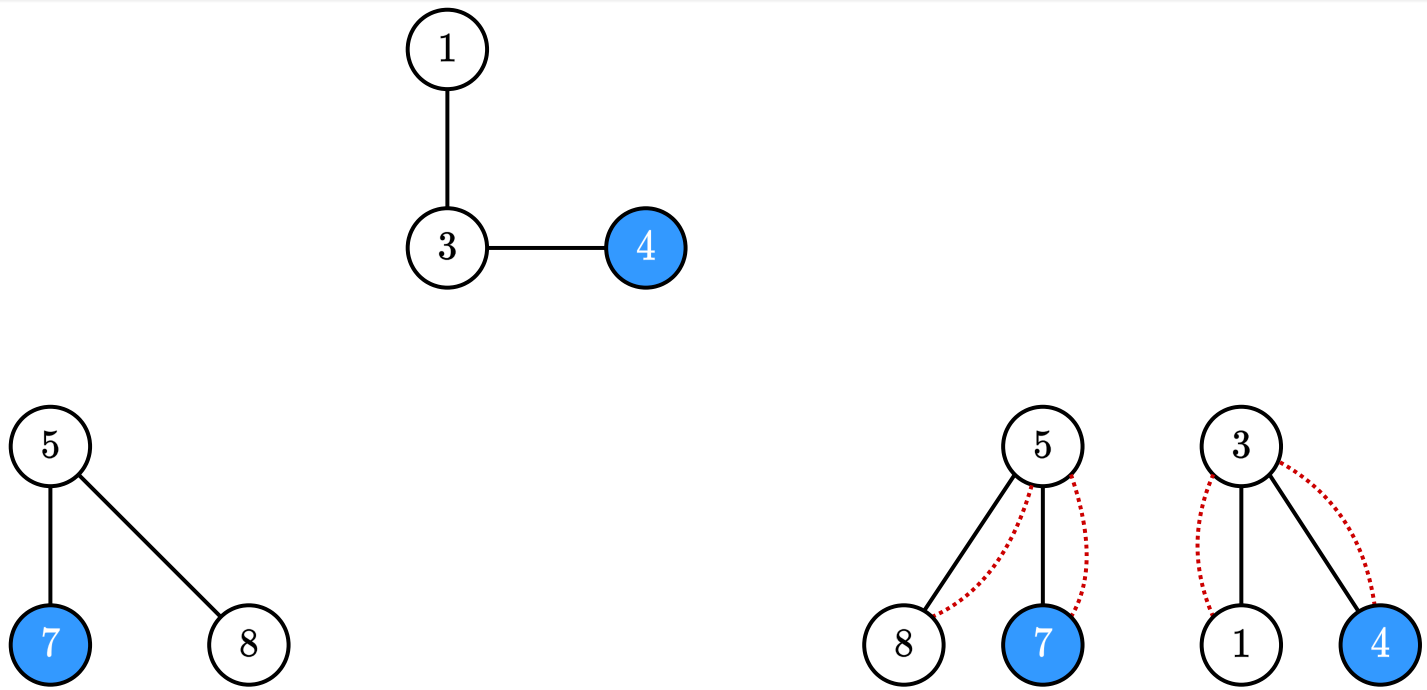

Exploiting sparsity

Idea: exploit *sparsity* of graphs that arise the program.

In CFGs: each if/while-node has 2 outgoing edges, others have only 1.

In call graphs: we don't expect a function to call a lot of other functions.

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In call graphs: we don't expect a function to call a lot of other functions.

Magic formula

For any problem involving graphs:

$$\text{input graphs are sparse} \wedge \text{problem is simpler on sparse graphs} \implies \text{faster algorithms}$$

Applying the formula

Applying the magic formula on CFGs:

- CFGs have small treewidth, shown by Thorup (Inf. Comput.'98) [4].
- Chattarjee et al. (ESOP'20) used this to develop an algorithm with:
  - Preprocessing in $O(n \cdot D^3)$ time and $O(\lceil \frac{D}{\lg n} \rceil)$ time per query.
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Applying the magic formula on call graphs:

- Call graphs have small treedepth.
- *Experimentally:* We analyzed program from DaCapo,
  - Avg. # of functions = 803.1.
  - Avg. treedepth = 43.8.
  - Max. treedepth = 135.
- *Intuition:* functions are developed in chronological order, each function uses a small subset of previously-developed functions as a library.
- In general, we expect treedepth to scale very slowly with program size.

Extra input

Our algorithm uses both parameters, so we'll assume:

- For every function $f \in F$, we are given a TD T_f of f's CFG G_f, and T_f has small width $\texttt{tw}$.
- We are given a POT T over the call graph C, and T has small depth $\texttt{td}$.

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We know that such T_f's and T exist, but how to compute them?

- This is NP-hard in general.
- There are efficient algorithms that compute a TD/POT if its width/depth is small [5, 6].
- In our experiments, we use heuristic solvers.

Where we are

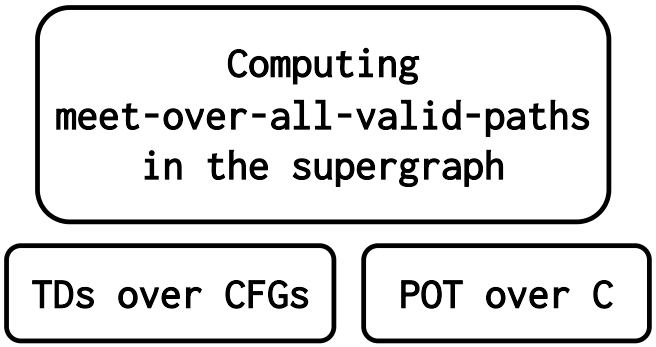
Computing
meet-over-all-valid-paths
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Table of Contents

IFDS problems

IFDS problem

Input: $\langle G = (V, E), D, \{M_e\}_{e \in E} \rangle$ queries of the form $\langle u_1, D_1, u_2 \rangle$.

Output: for each query $\langle u_1, D_1, u_2 \rangle$, return:

$$\texttt{MIVP}(u_1, D_1, u_2).$$

Simplification via distributivity

- Flow functions are distributive:
  $M_e(\{d_1, \ldots, d_k\}) = M_e(\emptyset) \cup M_e(\{d_1\}) \cup \cdots \cup M_e(\{d_k\})$
  $\Longrightarrow$ It suffices to know $M_e(\emptyset)$ and $M_e(\{d\})$ for every $d \in D$.

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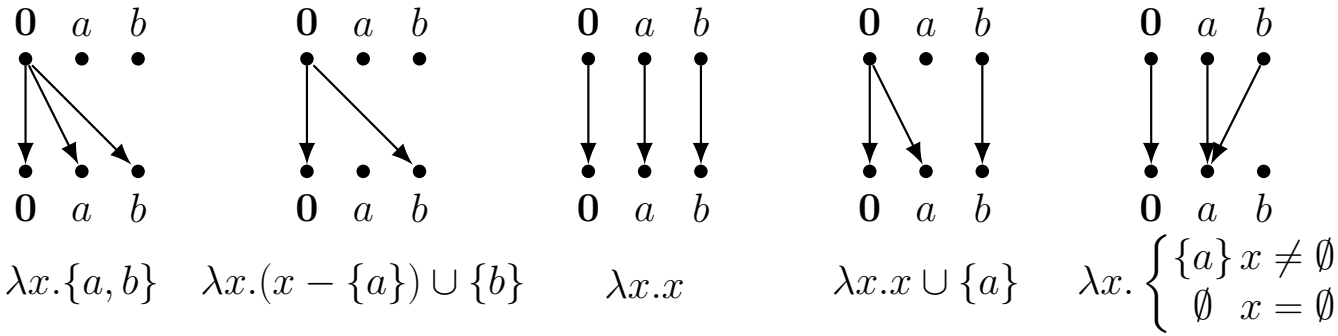
- Can represent a function with a bipartite graph of sides $D^* := D \cup \{\mathbf{0}\}$:



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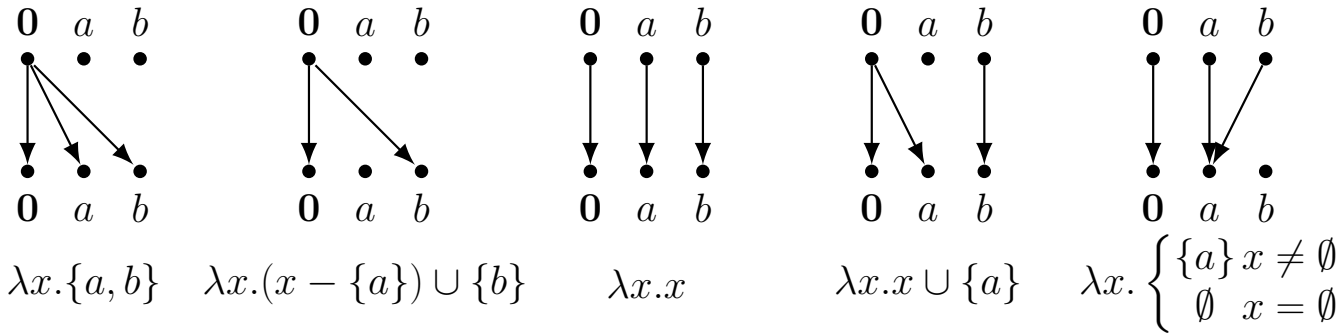
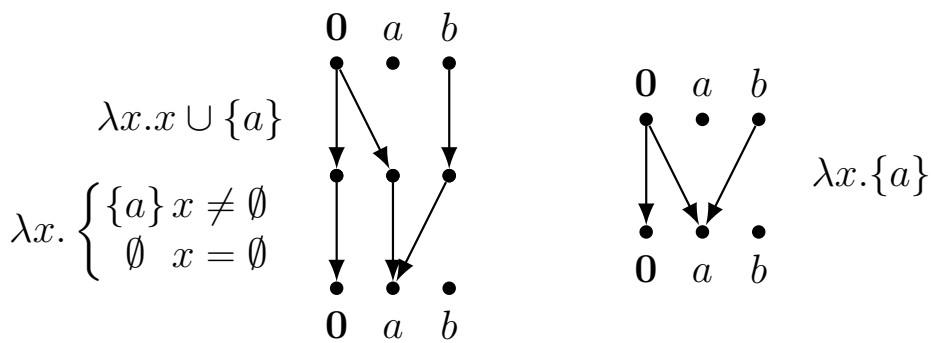
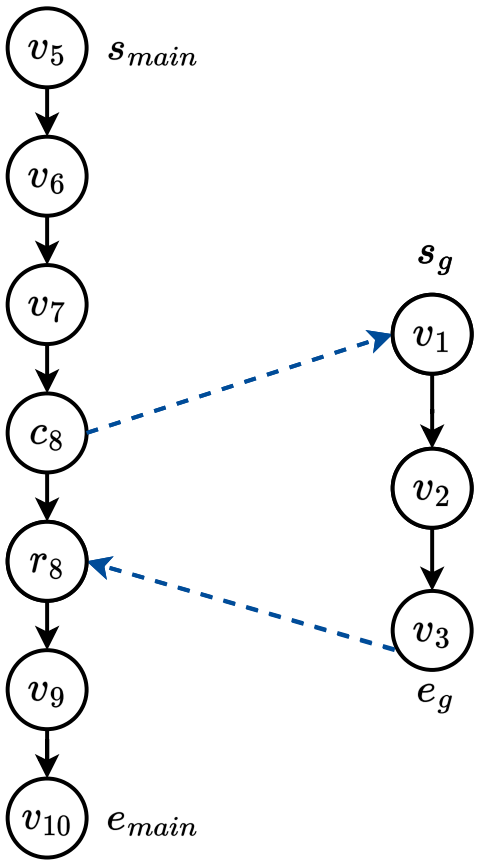


- Compositions of functions $\equiv$ reachability of their representation:



Exploded supergraphs

Replace each edge in a supergraph $G = (V, E)$ with their graph representation, which gives an exploded supergraph $G = \left(V \times D^*, \overline{E}\right)$:



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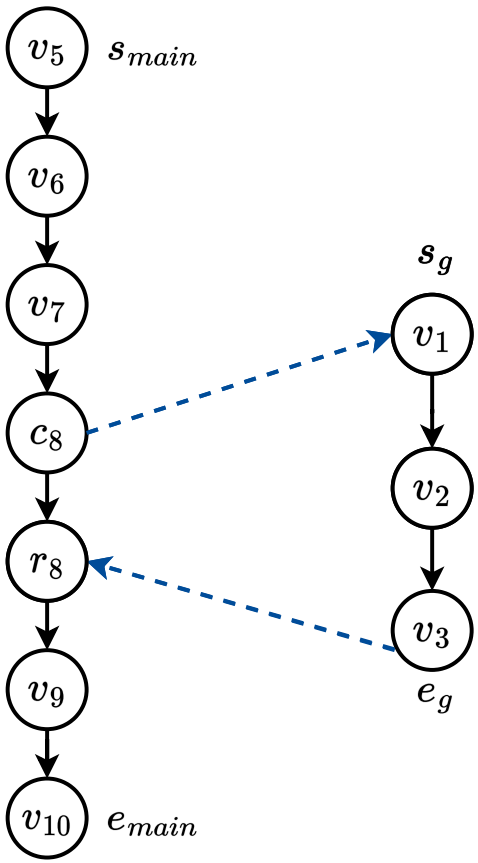

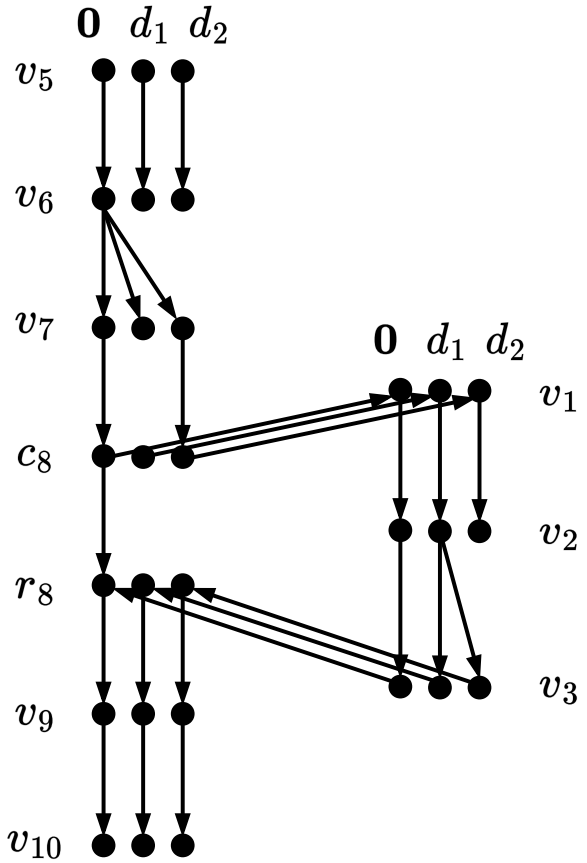

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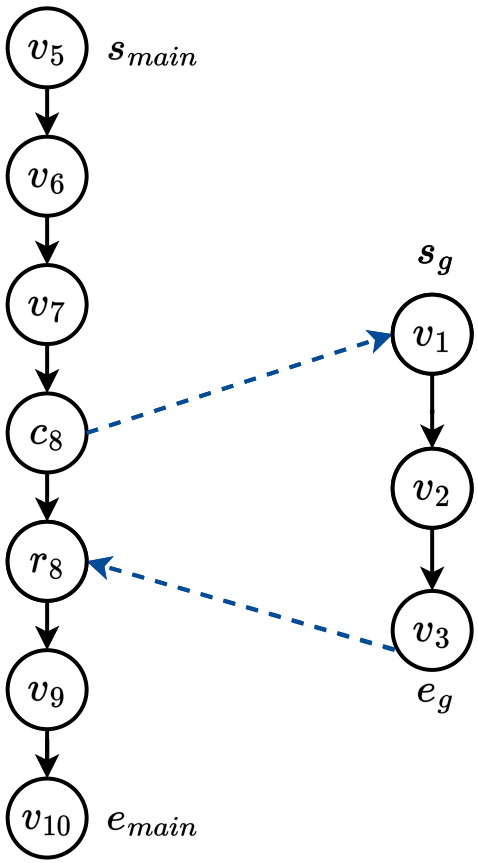
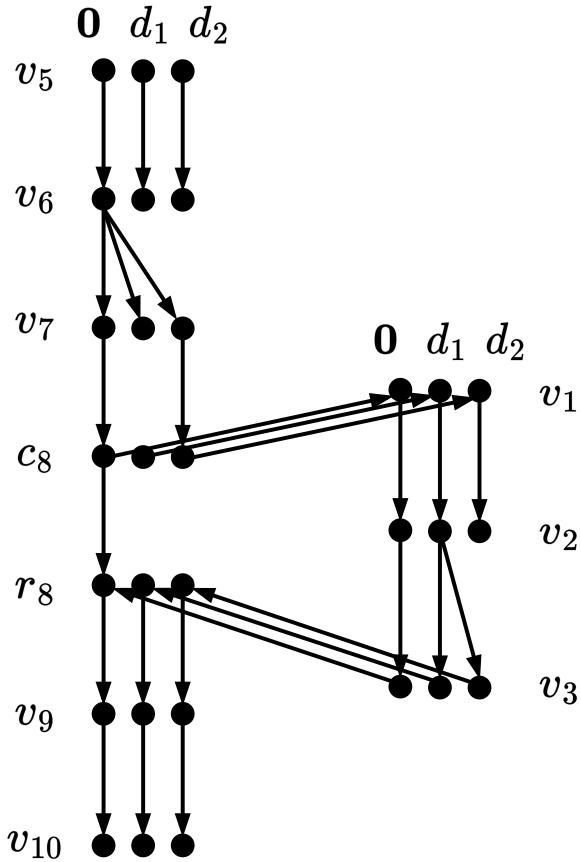


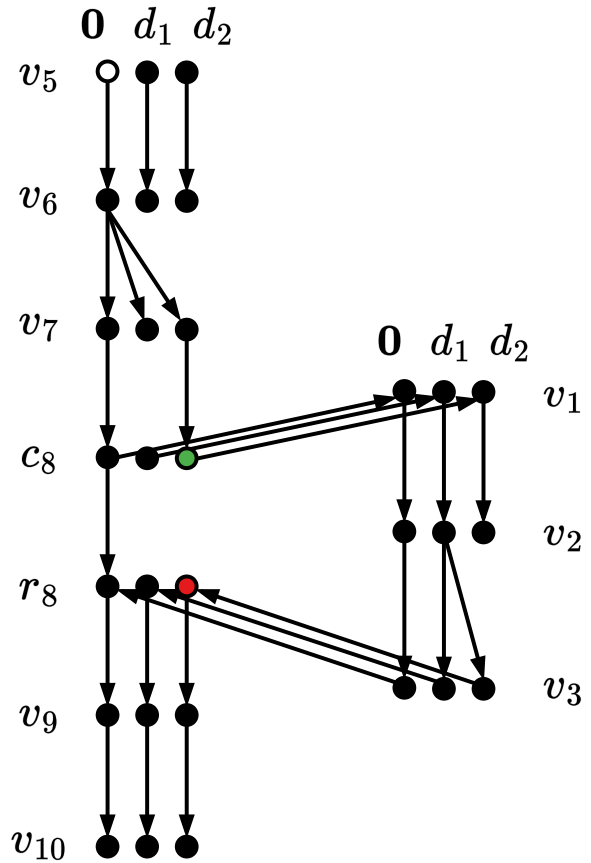


$\mathtt{Q}((u_1, d_1), (u_2, d_2)) := 1$ iff there is an IVP from (u_1, d_1) to (u_2, d_2) in $\overline{G}$.

Exploded supergraphs: example

- $d_2 =$ "b may be null"
- $\mathtt{Q}((v_5, \mathbf{0}), (c_8, d_2)) = 1 \implies$ b may be null after line 7.
- $\mathtt{Q}((v_5, \mathbf{0}), (r_8, d_2)) = 0 \implies$ b is not null after returning from call to g.

```
1  void g(int *&a, int *&b) {
2      b = a;
3  }
4
5  int main() {
6      int *a, *b;
7      a = new int(42);
8      g(a, b);
9      *b = 0;
10 }
```



IFDS problems

Simpler problem: checking existence of an IVP in $\overline{G}$.

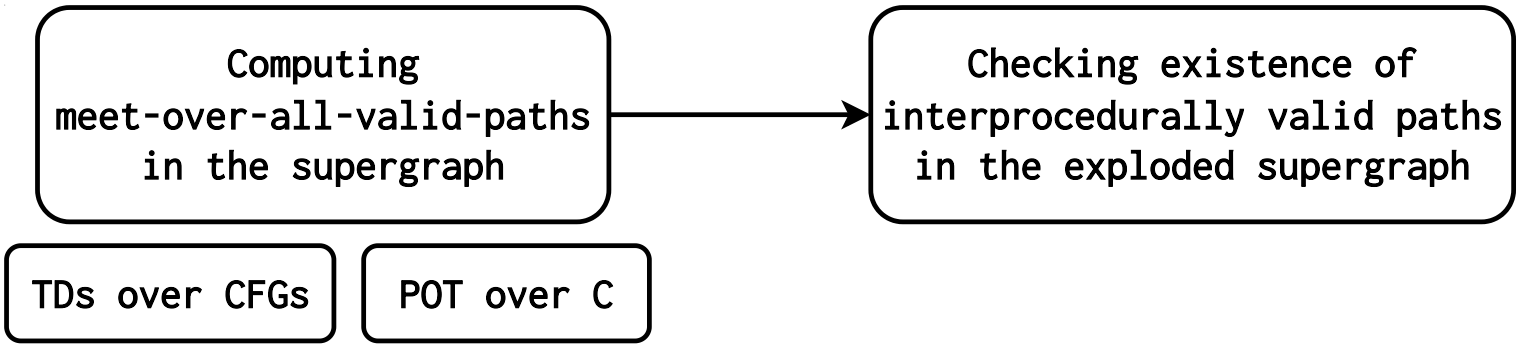
IFDS problem #2

Input: $\langle \overline{G} \rangle$ and queries of the form $\langle (u_1, d_1), (u_2, d_2) \rangle$.

Output: for each query $\langle (u_1, d_1), (u_2, d_2) \rangle$, return:

$$\mathtt{Q}((u_1, d_1), (u_2, d_2)).$$

Where we are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Same-context paths

A same-context path (SCP) in $G/\overline{G}$ is a special IVP that keeps the call-stack intact.

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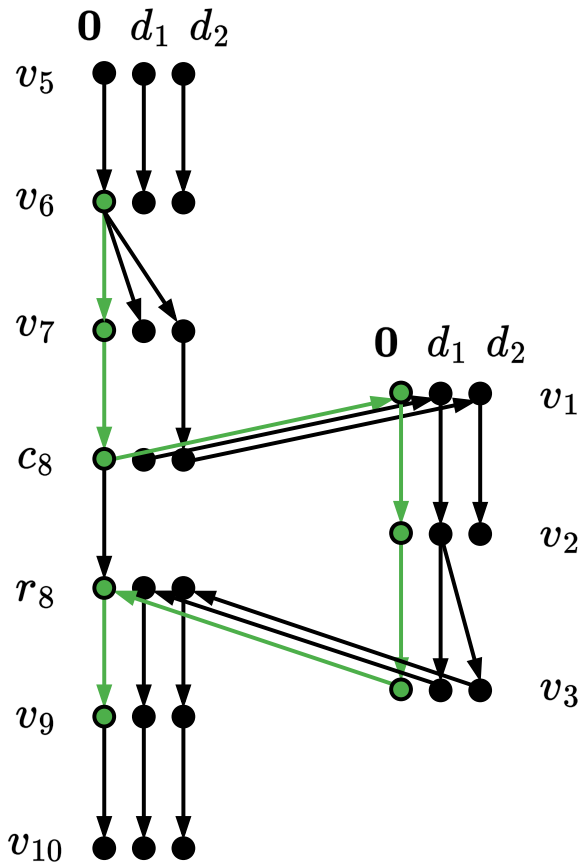

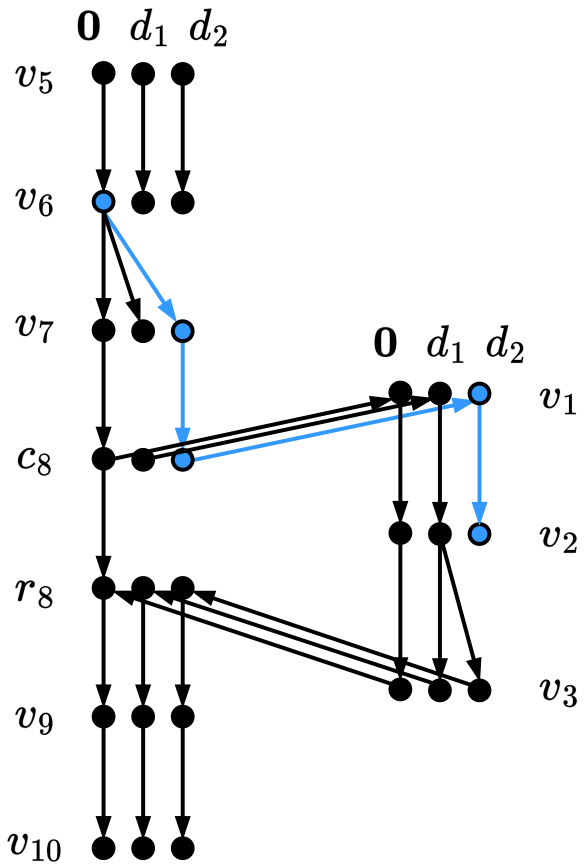

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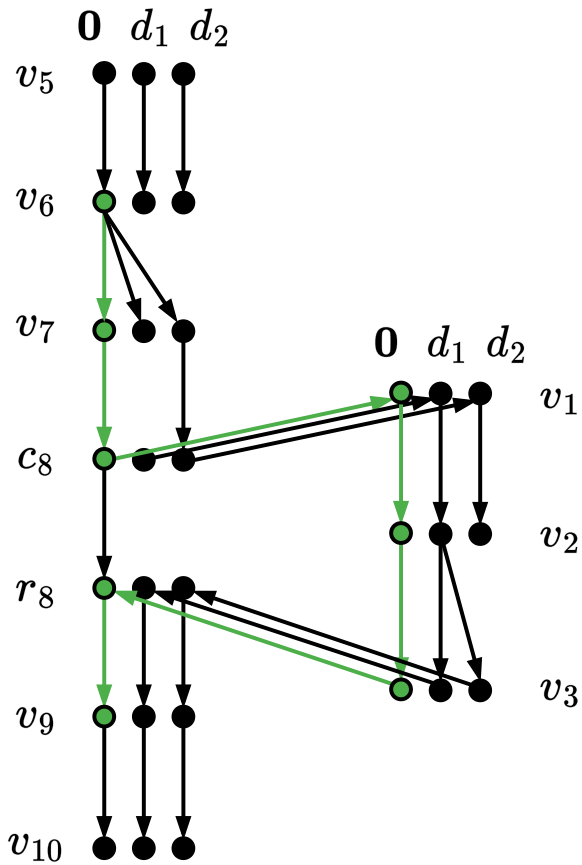

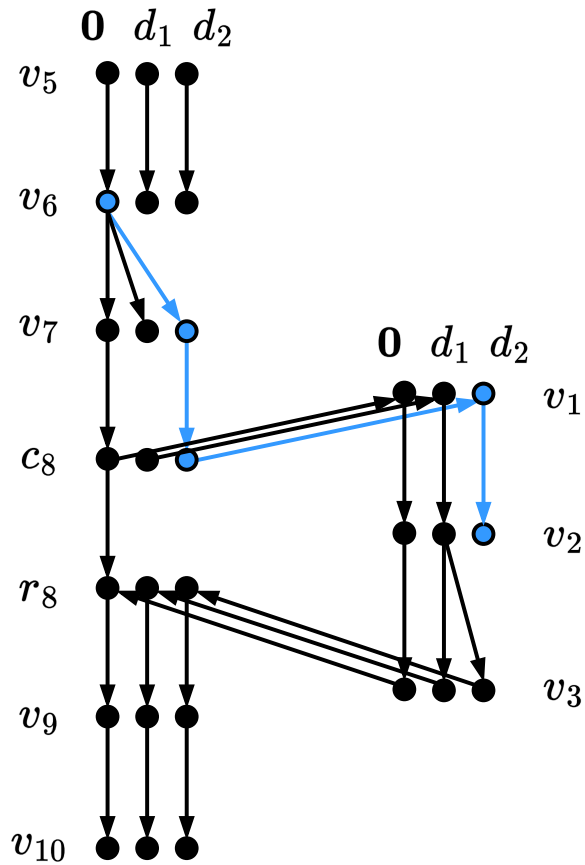

$\texttt{SCQ}((u_1, d_1), (u_2, d_2)) := 1$ iff there is an SCP from (u_1, d_1) to (u_2, d_2) in $\overline{G}$.

Canonical partition

Idea: consider an IVP π in $\overline{G}$, there are two types of call nodes in π:

- Temporary calls: calls c with a corresponding return node r later in π.
- Persistent calls: no corresponding return.

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Canonical partition

π can always be written as:

$$\pi = \Sigma_1 \cdot c_1 \cdot \Sigma_2 \cdot c_2 \cdots \Sigma_k \cdot c_k \cdot \Sigma_{k+1}$$

Where $c_1, \ldots, c_k$ are the *persistent* calls in π.

Canonical partition

$$\pi = (\Sigma_1 \cdot c_1) \cdot (\Sigma_2 \cdot c_2) \cdots (\Sigma_k \cdot c_k) \cdot \Sigma_{k+1}$$

Assumption: suppose π begins and ends at some start-node.

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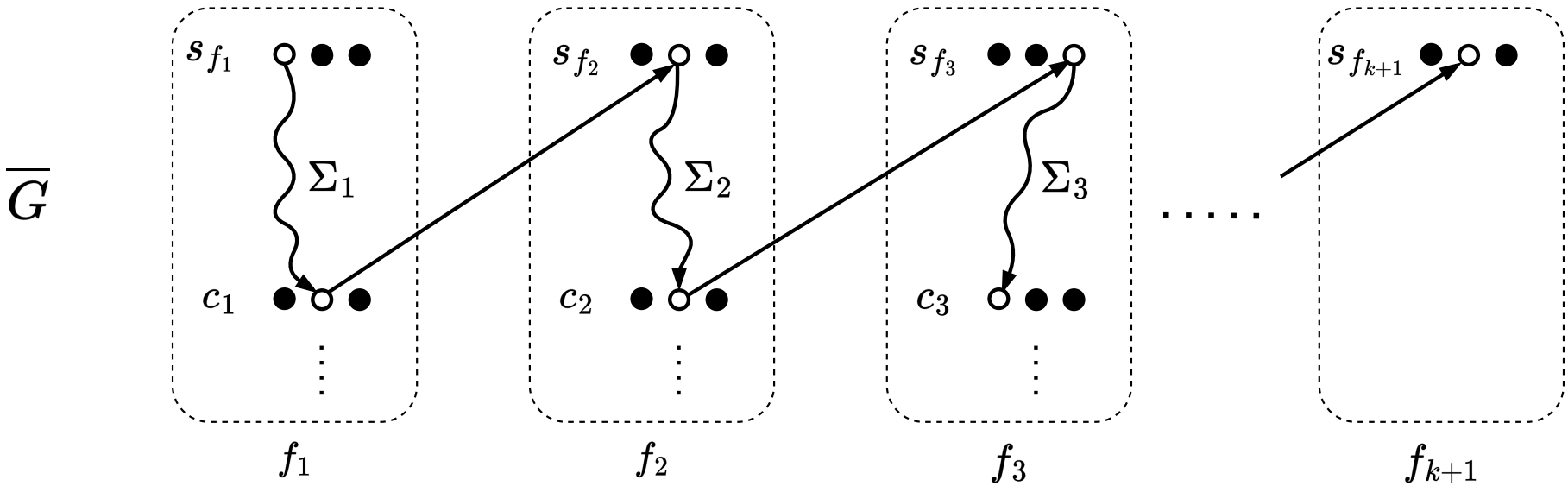
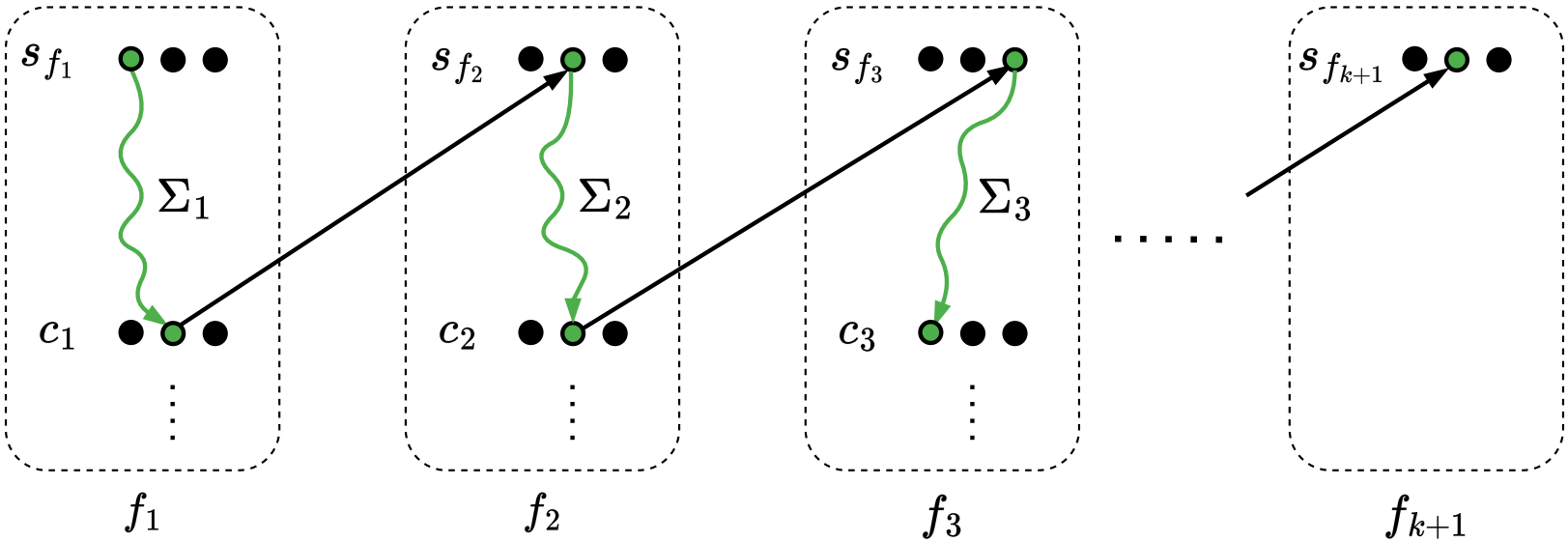


Canonical partition

$$\pi = (\Sigma_1 \cdot c_1) \cdot (\Sigma_2 \cdot c_2) \cdots (\Sigma_k \cdot c_k) \cdot \Sigma_{k+1}$$

Observation #1: each $\Sigma_i \cdot c_i$ is a same-context path.

$\overline{G}$



Canonical partition

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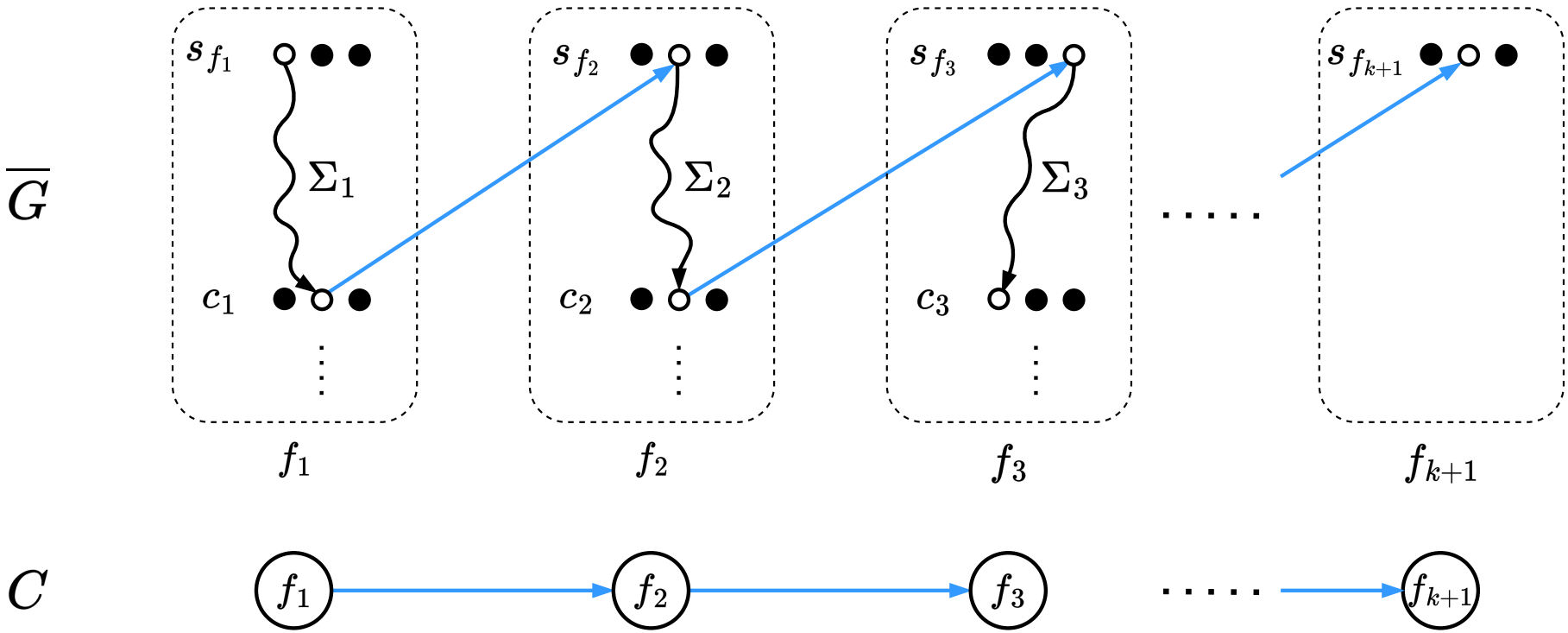
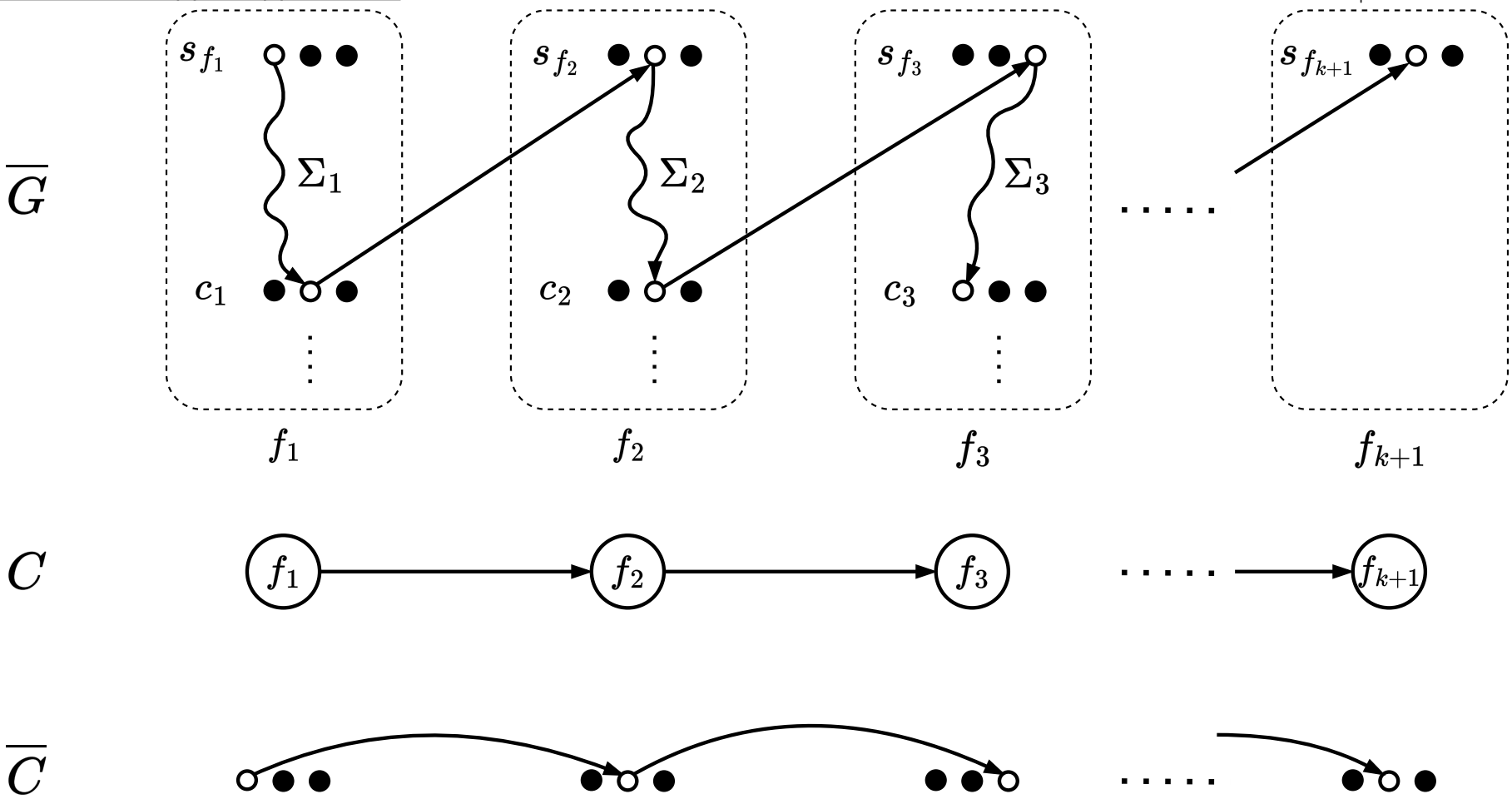
Observation #2: c_i calls $f_{i+1} \implies (f_i, f_{i+1})$ is an edge of the call graph.



Canonical partition

$$\pi = (\Sigma_1 \cdot c_1) \cdot (\Sigma_2 \cdot c_2) \cdots (\Sigma_k \cdot c_k) \cdot \Sigma_{k+1}$$

Exploded call graph $\overline{C}$: each edge abstracts a segment $\Sigma_i \cdot c_i \cdot s_{f_{i+1}}$.



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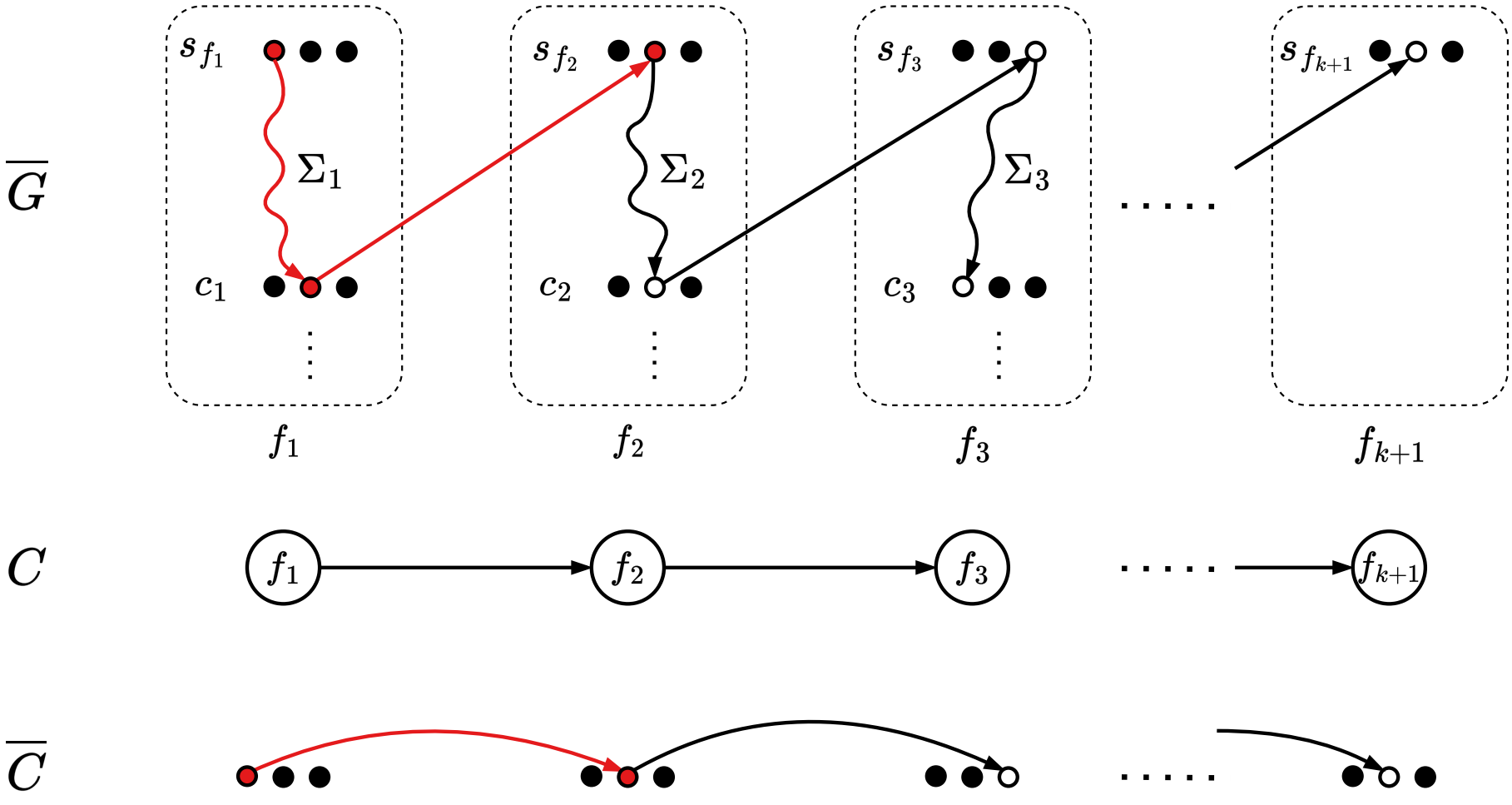

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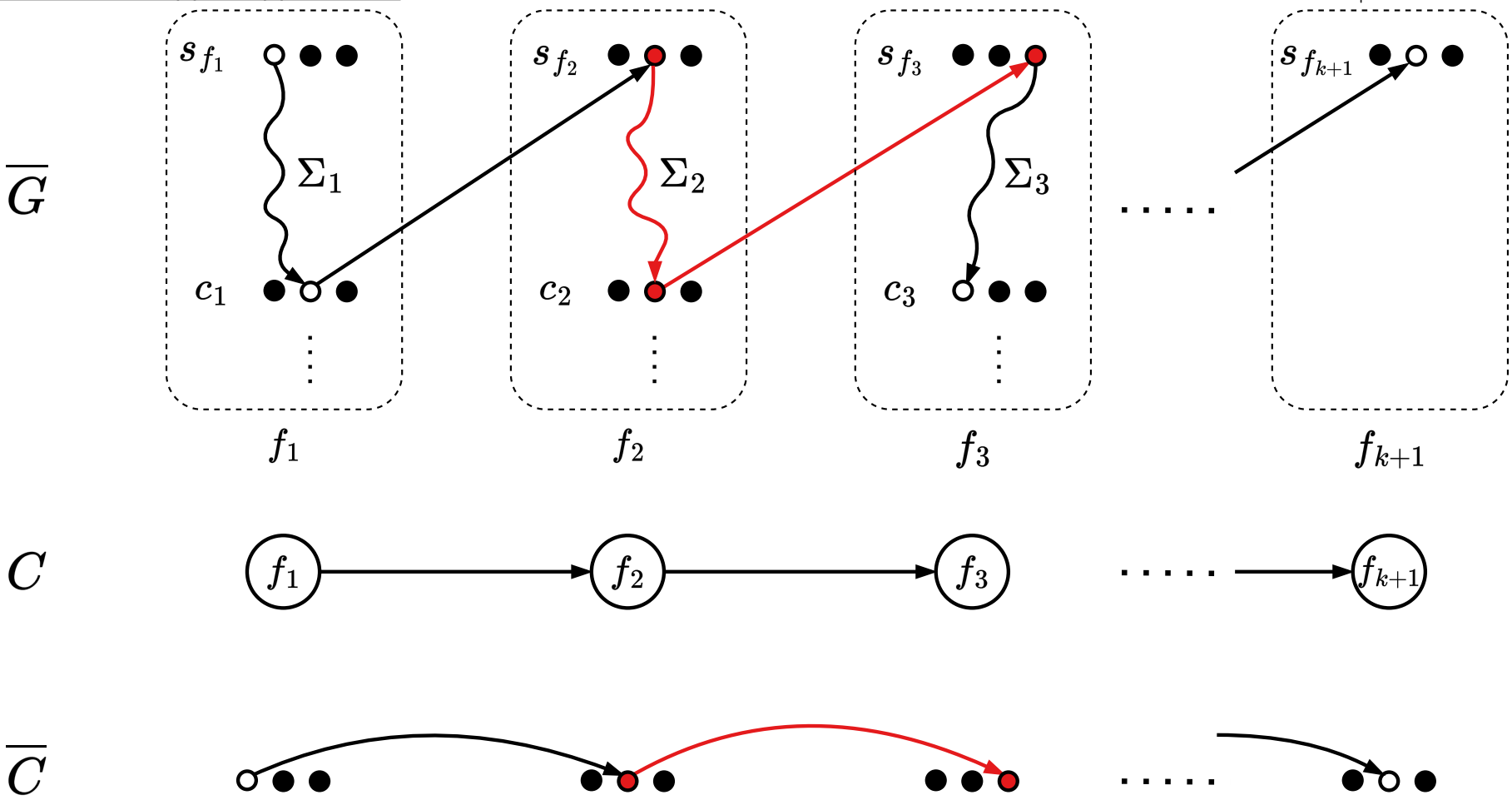

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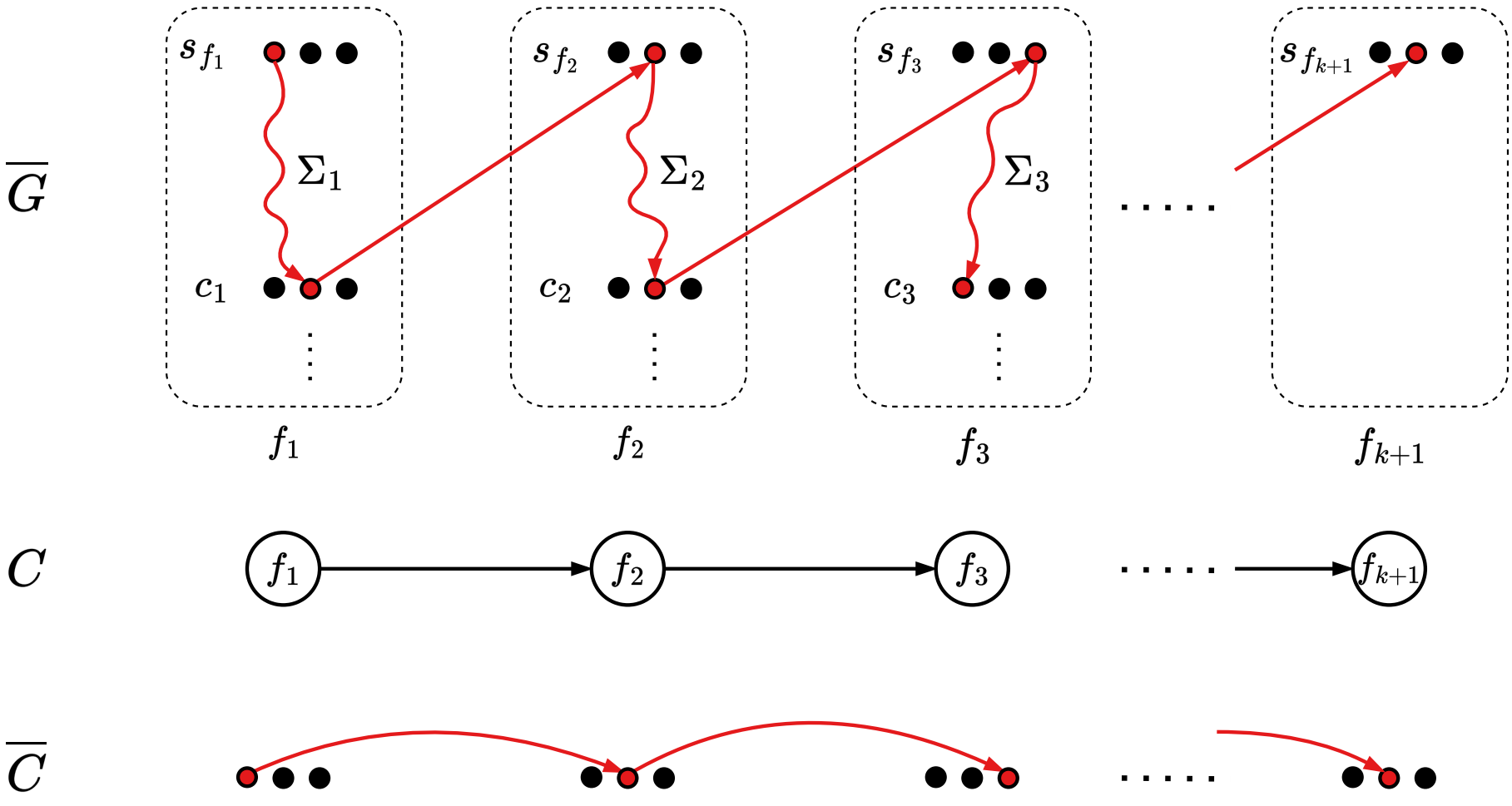
Existence of $\pi \implies$ existence of a path from $(f_1, \cdot)$ to $(f_{k+1}, \cdot)$ in $\overline{C}$



Exploded call graphs

For a call graph $C = (F, E_C)$, an exploded call graph $\overline{C} = (F \times D^*, \overline{E_C})$ has $((f_1, d_1), (f_2, d_2)) \in \overline{E_C}$ iff there is a $(c, d_3) \in V_f \times D^*$ s.t.

- $\texttt{SCQ}((s_{f_1}, d_1), (c, d_3))$
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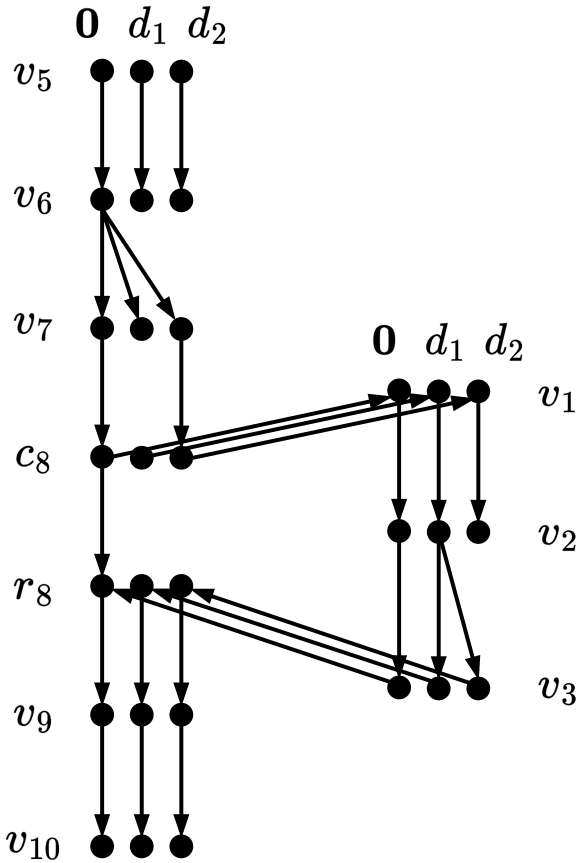

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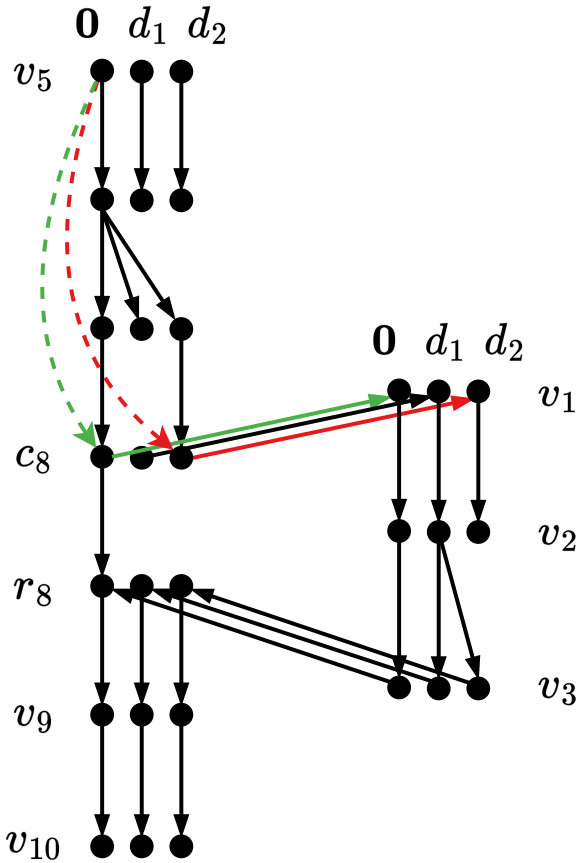

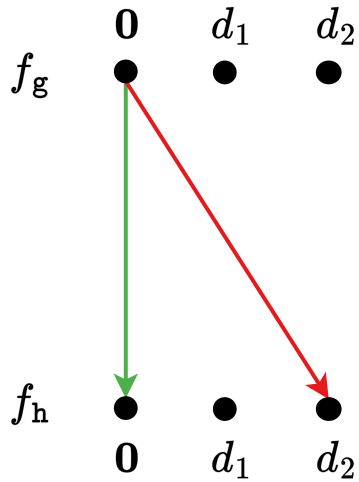

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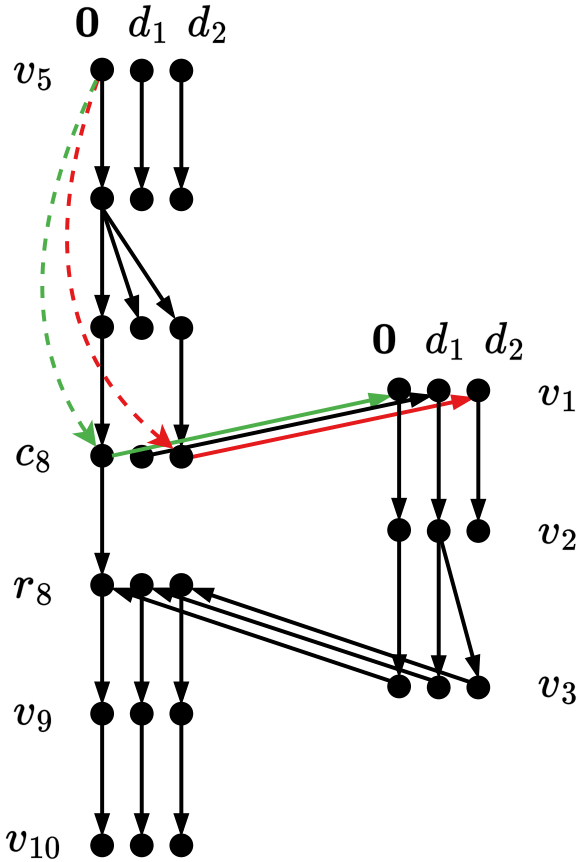
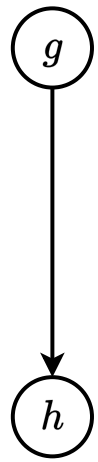
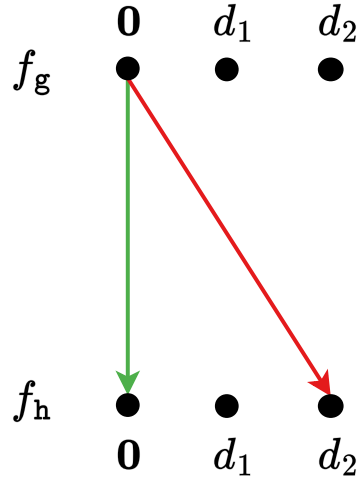






$$\forall \langle (s_{f_u}, d_1), (s_{f_v}, d_2) \rangle, \quad \mathtt{Q}((s_{f_u}, d_1), (s_{f_v}, d_2)) = (f_u, d_1) \rightsquigarrow_{\overline{C}} (f_v, d_2).$$

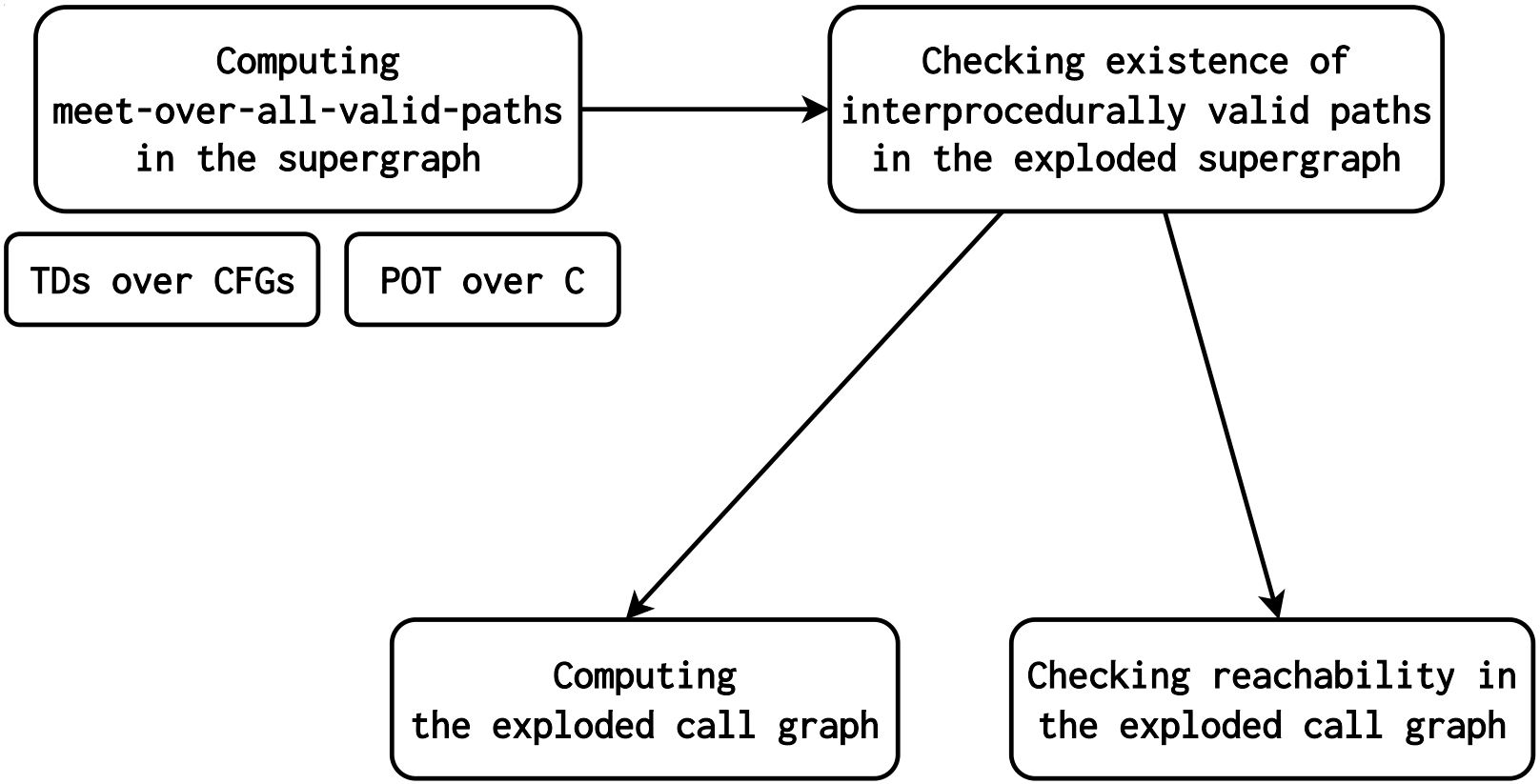
Exploded call graph

We now have two subproblems to answer queries of the form

$$\langle (s_{f_u}, d_1), (s_{f_v}, d_2) \rangle,$$

1. Computing the exploded call graph $\overline{C}$.
2. Answering reachability queries on $\overline{C}$.

Where we are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Computing $\overline{C}$

Recall, $((f_1, d_1), (f_2, d_2)) \in \overline{E_C}$ iff there is a $(c, d_3) \in V_f \times D^*$ s.t

- $\texttt{SCQ}((s_{f_1}, d_1), (c, d_3))$
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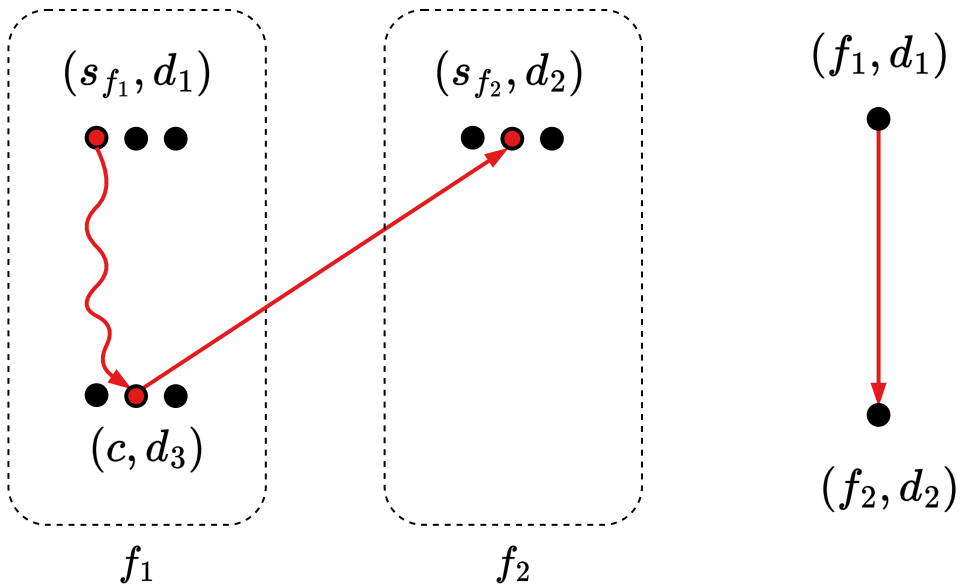
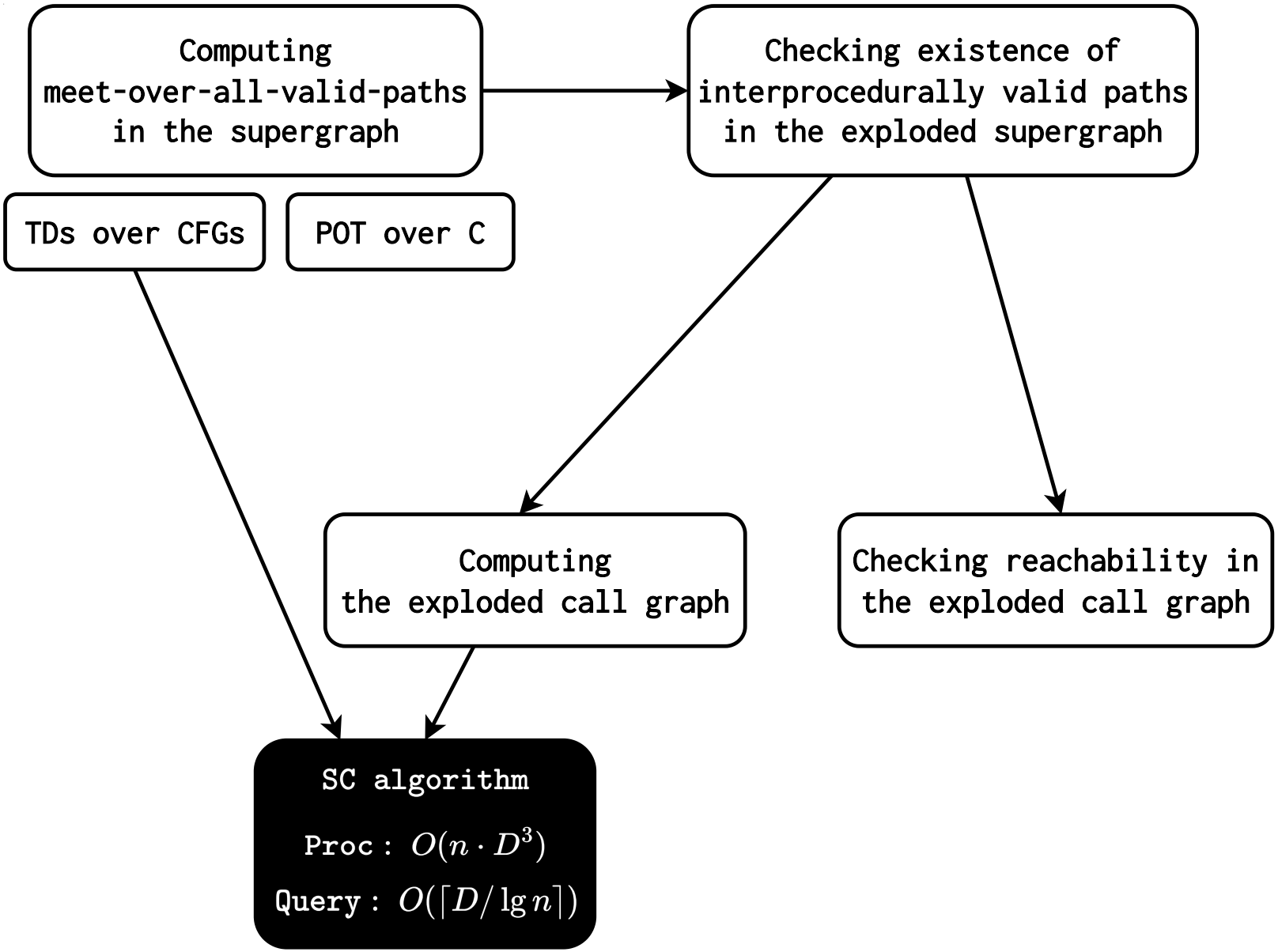


Algorithm:

1. Iterate over all possible $((f_1, d_1), (c, d_3))$.
2. Invoke Chatterjee's algorithm to compute $\texttt{SCQ}((s_{f_1}, d_1), (c, d_3))$.
3. If it returns 1, add the corresponding $((f_1, d_1), (f_2, d_2))$ to $\overline{C}$.

Where we are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Computing
the exploded call graph
Checking reachability in
the exploded call graph
SC algorithm
Proc : $O(n \cdot D^3)$
Query : $O(\lceil D/\lg n \rceil)$

Checking reachability in $\overline{C}$

Reachability on $\overline{C}$

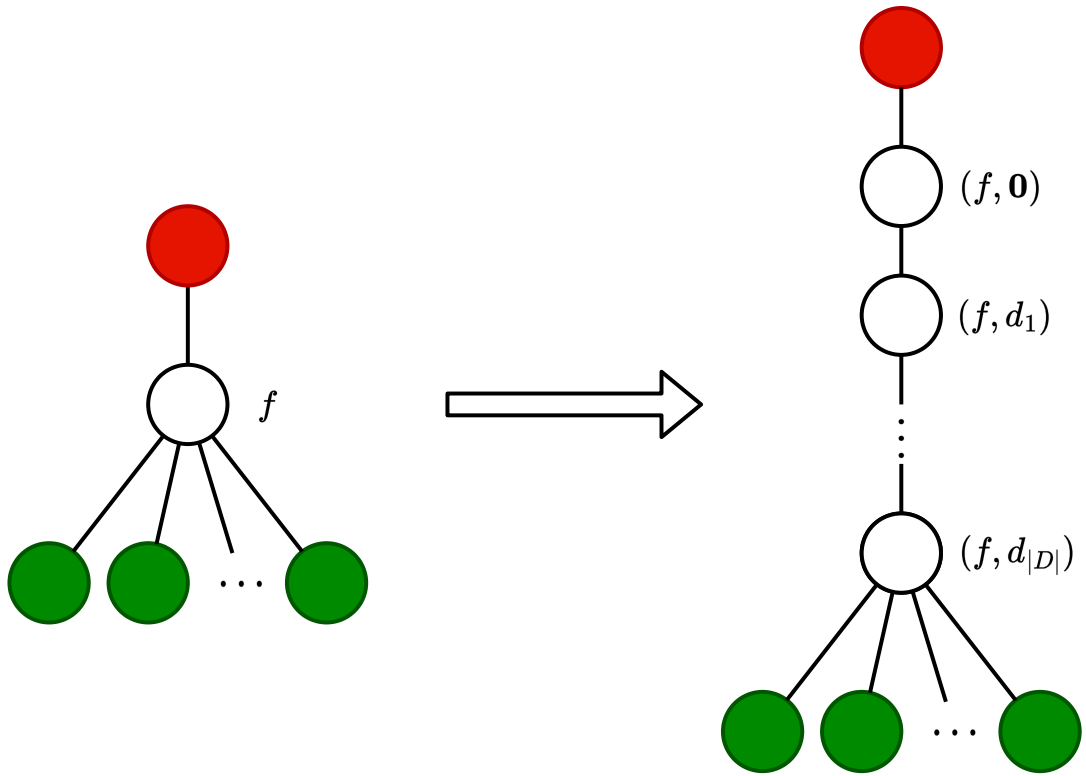
Input: $\langle \overline{C} \rangle$ and queries of the form $\langle (f_u, d_1), (f_v, d_2) \rangle$.

Output: for each query $\langle (f_u, d_1), (f_v, d_2) \rangle$, return:

$$(f_u, d_1) \rightsquigarrow_{\overline{C}} (f_v, d_2).$$

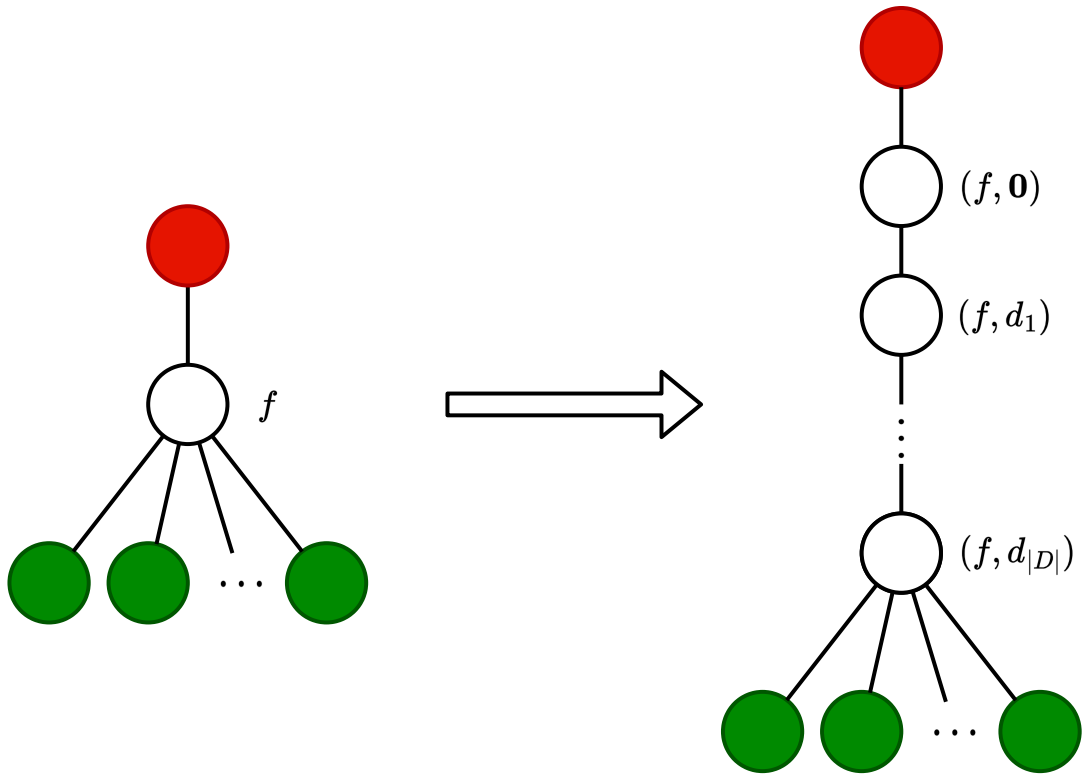
Checking reachability in $\overline{C}$

We have a POT T over C : explode it into a POT $\overline{T}$ over $\overline{C}$.



Checking reachability in $\overline{C}$

We have a POT T over C : explode it into a POT $\overline{T}$ over $\overline{C}$.



T has depth $\mathtt{td} \implies \overline{T}$ has depth $\mathtt{td} \cdot D$, which is still small.

Exploiting treedepth

Reachability on $\overline{C}$ using POT $\overline{T}$

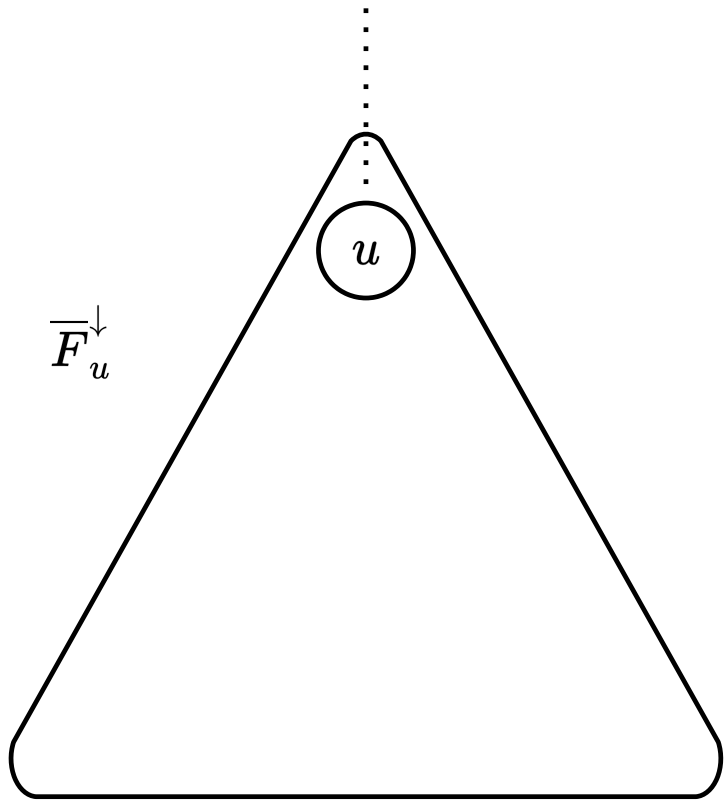
Input: $\langle \overline{C}, \overline{T} \rangle$ and queries of the form $\langle (f_u, d_1), (f_v, d_2) \rangle$.

Output: for each query $\langle (f_u, d_1), (f_v, d_2) \rangle$, return:

$$(f_u, d_1) \rightsquigarrow_{\overline{C}} (f_v, d_2).$$

Reachability on $\overline{C}$ using POT $\overline{T}$

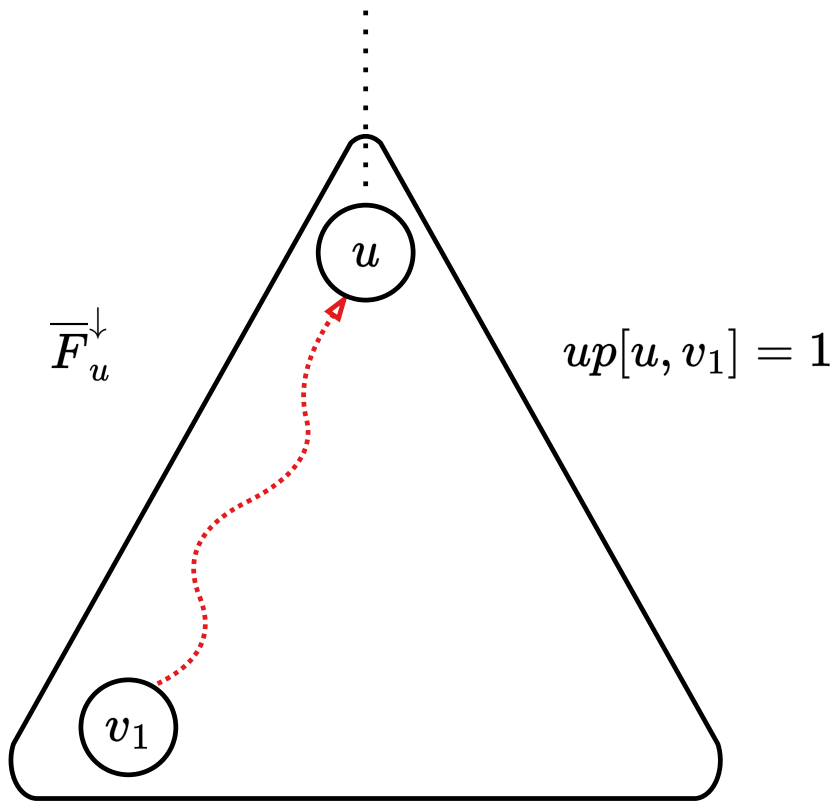
Let $\overline{F}_u^{\downarrow}$ be the set of descendants of u in $\overline{T}$.



Reachability on $\overline{C}$ using POT $\overline{T}$

For every u and every descendant v of it, define:

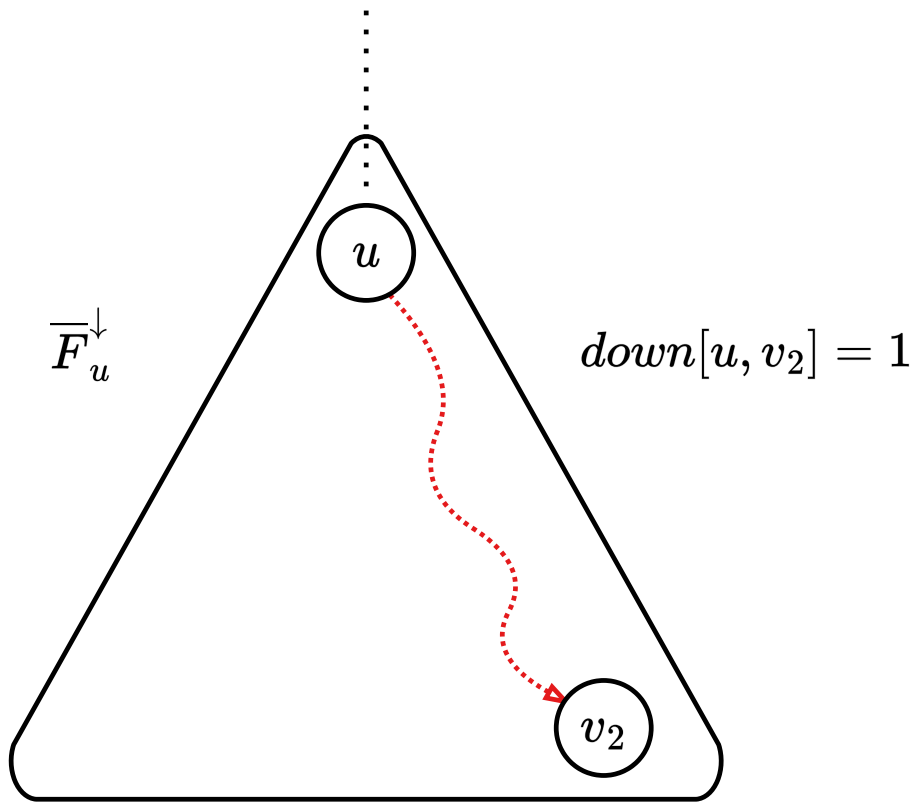
$$up[u, v] := \begin{cases} 1 & \text{there is a path from } v \text{ to } u \text{ in } \overline{C}[\overline{F}_u^{\downarrow}] \\ 0 & \text{otherwise} \end{cases}.$$



Reachability on $\overline{C}$ using POT $\overline{T}$

For every u and every descendant v of it, define:

$$down[u, v] := \begin{cases} 1 & \text{there is a path from } u \text{ to } v \text{ in } \overline{C}[\overline{F}_u^{\downarrow}] \\ 0 & \text{otherwise} \end{cases}.$$



Reachability on $\overline{C}$ using POT $\overline{T}$: preprocessing

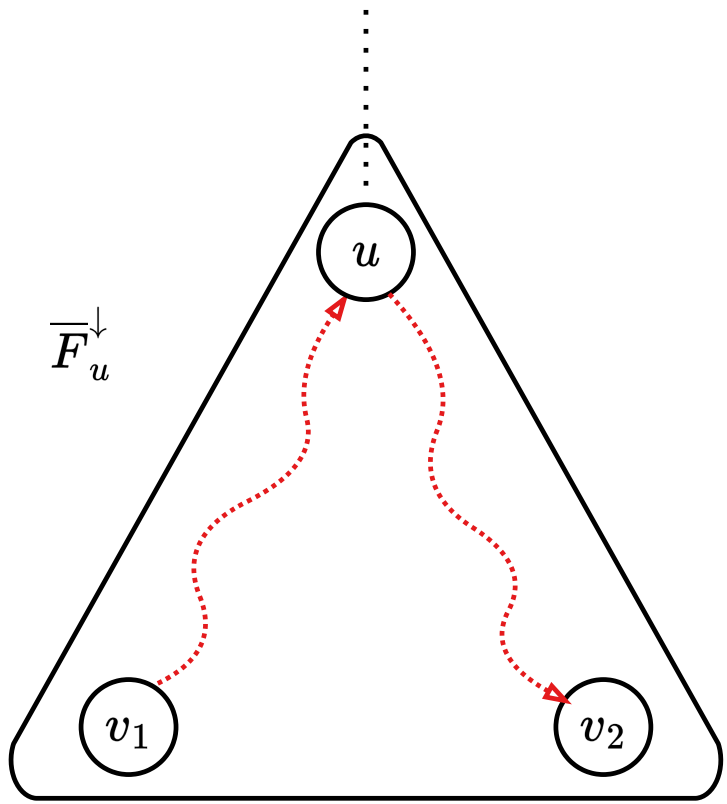
Preprocessing: compute *up* and *down*.

Reachability on $\overline{C}$ using POT $\overline{T}$: preprocessing

Preprocessing: compute *up* and *down*.

— $down[u, \cdot]$ is computed by a DFS from u, ignoring edges leaving $\overline{C}[\overline{F}_u^{\downarrow}]$.

— $up[u, \cdot]$ is similarly computed by reversing edges of $\overline{C}$.



Reachability on $\overline{C}$ using POT $\overline{T}$: preprocessing

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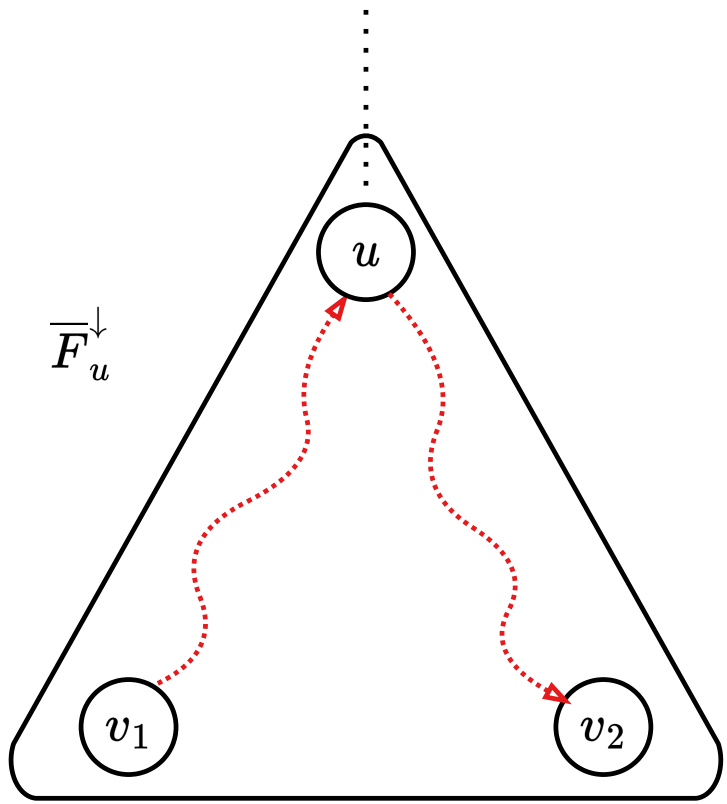
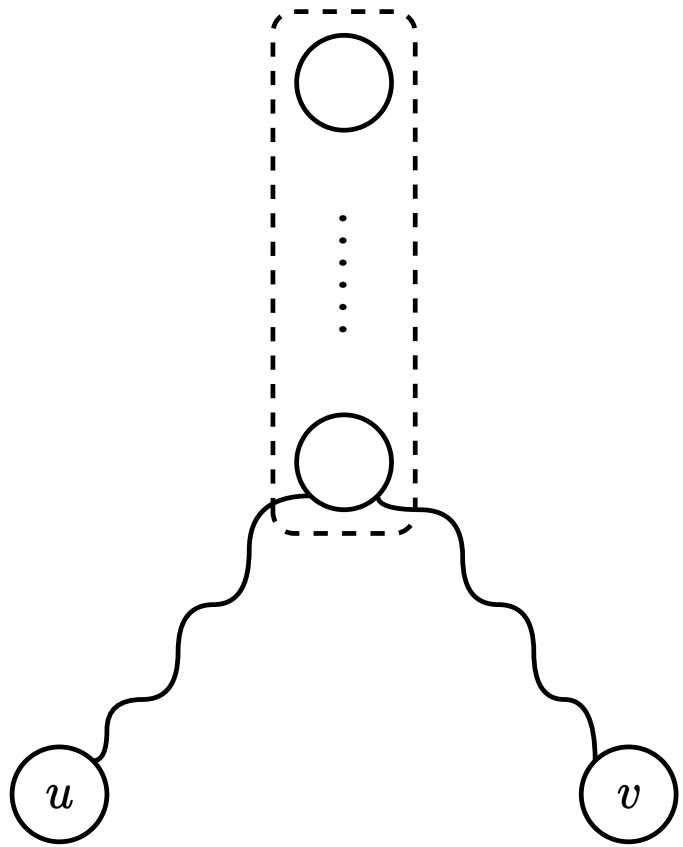


Each edge is traversed $O(\text{depth of } \overline{T}) = O(\mathtt{td} \cdot D)$ times.

$\Longrightarrow$ *up* and *down* can be computed in $O(n \cdot D^3 \cdot \mathtt{td})$ time.

Reachability on $\overline{C}$ using POT $\overline{T}$: queries

For any u, v in $\overline{C}$, let A be the set of their common ancestors in $\overline{T}$.

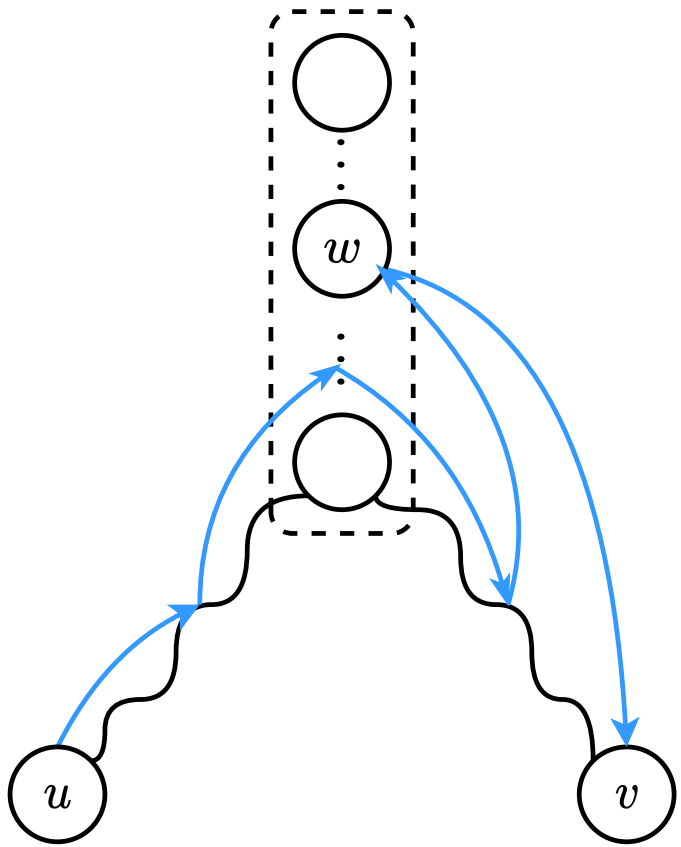


Reachability on $\overline{C}$ using POT $\overline{T}$: queries

By the cut property of POTs, any path ρ from u to v in $\overline{C}$ has:

$$\rho \cap A \neq \emptyset$$

.

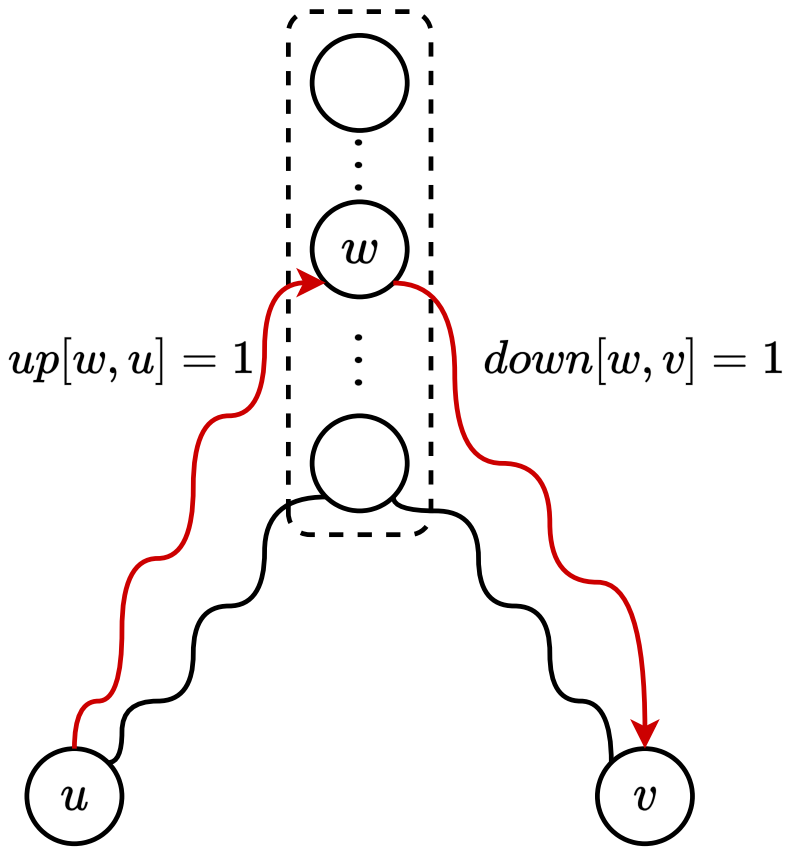


Reachability on $\overline{C}$ using POT $\overline{T}$: queries

Let $w \in A$ be the highest node in $\rho \cap A$.

We must have:

$$up[w, u] = 1 \wedge down[w, v] = 1.$$



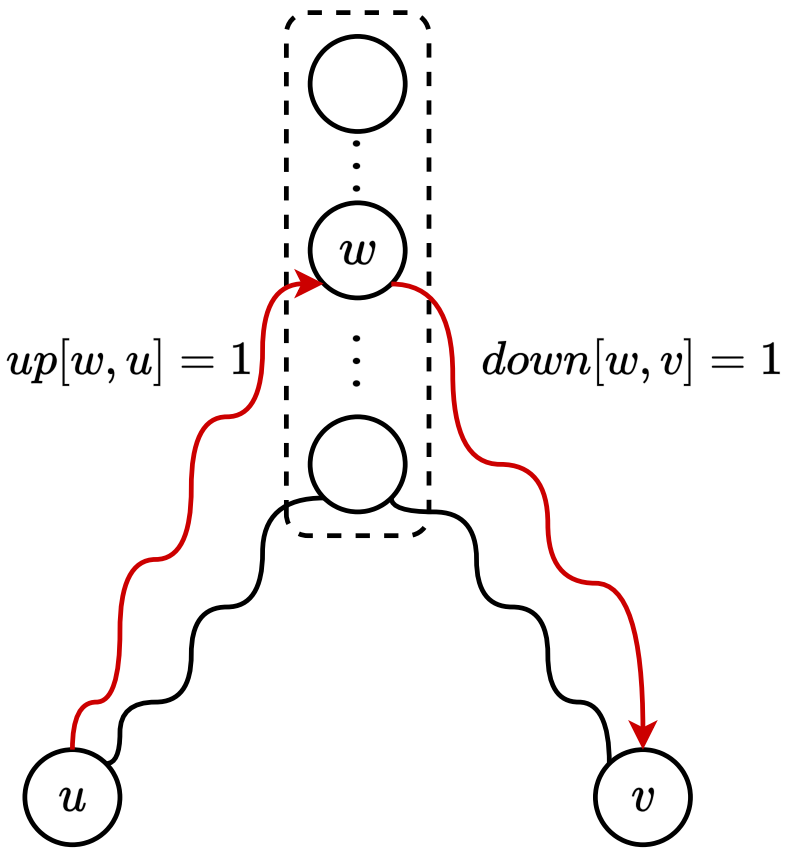
Reachability on $\overline{C}$ using POT $\overline{T}$: queries

$$u \rightsquigarrow_{\overline{C}} v \text{ iff } \exists w \in A \text{ s.t. } up[w,u] = 1 \wedge down[w,v] = 1.$$

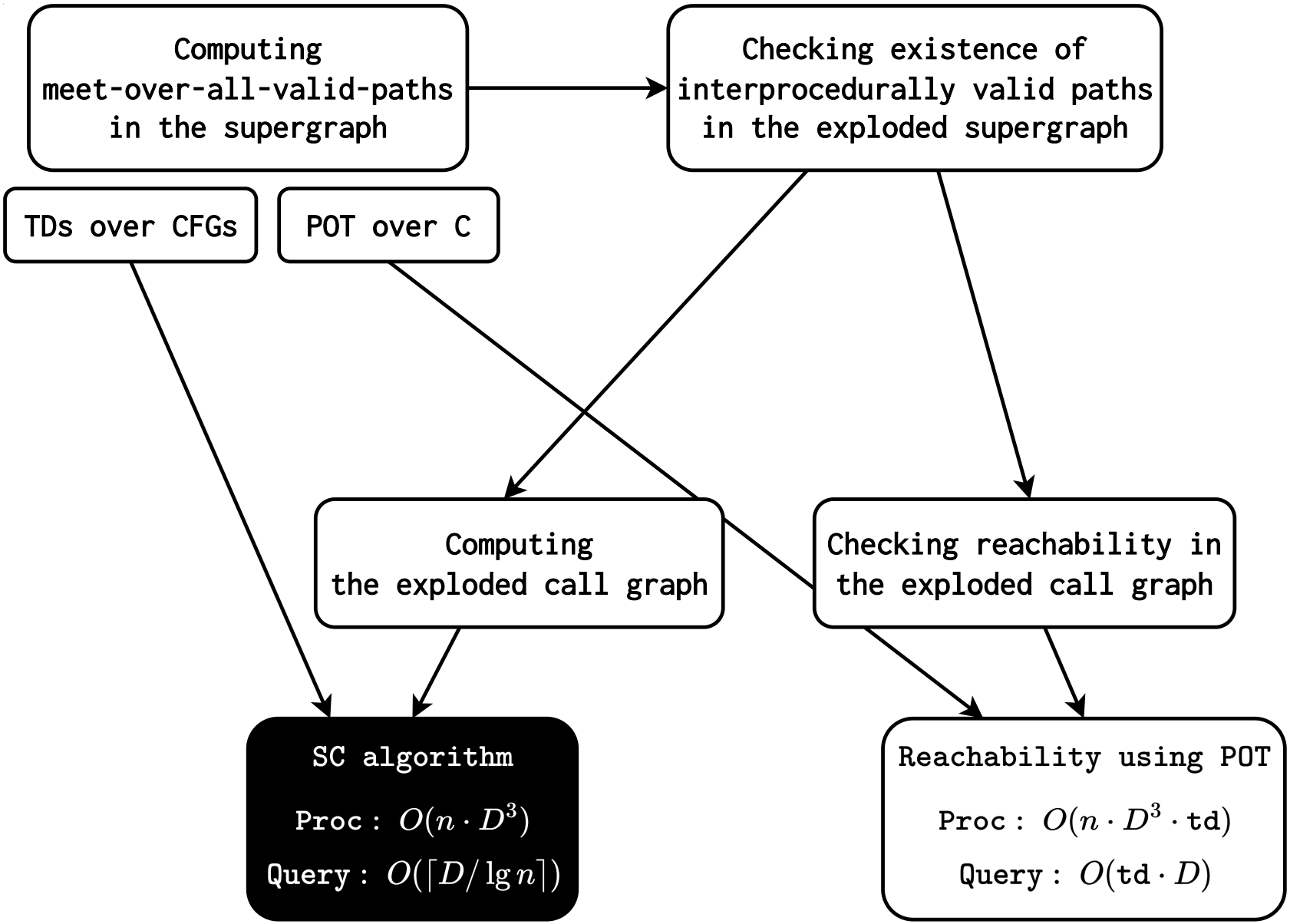
To answer a query $\langle u, v \rangle$: we iterate over w and check if

$$up[w,u] = 1 \wedge down[w,v] = 1$$

$\implies$ query time $O(\text{depth of } \overline{T}) = O(\texttt{td} \cdot D)$.



Where we are


Computing
meet-over-all-valid-paths
in the supergraph
Checking existence of
interprocedurally valid paths
in the exploded supergraph
TDs over CFGs
POT over C
Computing
the exploded call graph
Checking reachability in
the exploded call graph
SC algorithm
Proc : $O(n \cdot D^3)$
Query : $O(\lceil D / \lg n \rceil)$
Reachability using POT
Proc : $O(n \cdot D^3 \cdot \mathtt{td})$
Query : $O(\mathtt{td} \cdot D)$

Answering a general query on $\overline{G}$

$\overline{C}$ helps us compute $\mathtt{Q}((s_{f_u}, d_1), (s_{f_v}, d_2))$, which is a restricted form.

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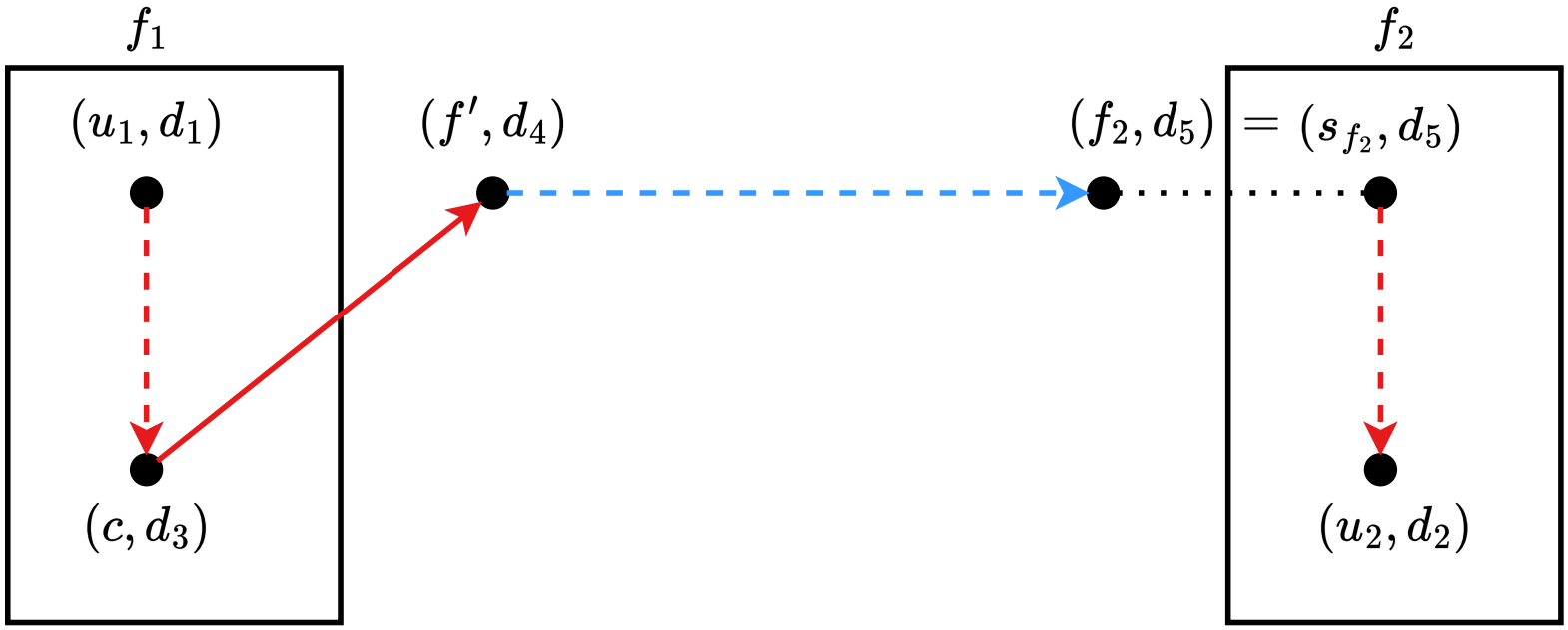


To compute $\mathtt{Q}((u_1, d_1), (u_2, d_2))$:

- Iterate over calls (c, d_3) in the same function as u_1.
- If (c, d_3) calls $(s_{f'}, d_4)$, perform:
  - *Same-context query:* check $\mathtt{SCQ}((u_1, d_1), (c, d_3))$.
  - *Reachability query on $\overline{C}$:* check $(f', d_4) \rightsquigarrow_{\overline{C}} (f_2, d_5)$, and
  - *Same-context query:* check $\mathtt{SCQ}((s_{f_2}, d_5), (u_2, d_2))$.

Answering a general query on $\overline{G}$

Done!

Runtime

- Preprocessing: $O(n \cdot D^3 \cdot \texttt{td})$.
- Query: $O(D^3 \cdot \texttt{td})$.

where:

- $n =\#$ lines in the program
- D number of possible data facts.
- $\texttt{td} =$ treedepth of the call graph.

Runtime

- Preprocessing: $O(n \cdot D^3 \cdot \texttt{td})$. $\leftarrow O(n)$ in practice.
- Query: $O(D^3 \cdot \texttt{td})$. $\leftarrow O(1)$ in practice.

where:

- $n = \#$ lines in the program
- D number of possible data facts.
- $\texttt{td} =$ treedepth of the call graph.

Table of Contents

Experiments: setup

- Ran the algorithm on real-world programs from DaCapo benchmarks.
- Extracted the CFGs and call graph using Soot.
- Used PACE solvers [7, 8] to compute:
  - TDs of the CFGs of small width.
  - POT over the call graph of small depth.
- On each benchmark we ran reachability, null-pointer, and possibly-uninitialized variables analyses.
- For a program of n lines, we generate n random queries.
- Ran each analysis on:
  - (`PARAM`) our algorithm,
  - (`IFDS`) standard IFDS algorithm [1], and
  - (`DEM`) its demand version [2].

  timing out at 10 minutes.

Experiments: results

Average/maximum are over 13 programs from DaCapo benchmarks.

- $|V|$ ($\approx$ lines of code):
  - Average: 22.7K.
  - Maximum: 58.5K.
- Number of functions:
  - Average: 803.1.
  - Maximum: 2028.
- Treewidth of CFGs:
  - Average: 9.1.
  - Maximum: 10.
- Treedepth of call graphs:
  - Average: 43.8.
  - Maximum: 135.

Experiments: reachability

Preprocessing:

- Average: 0.93s.
- Maximum: 1.53s.

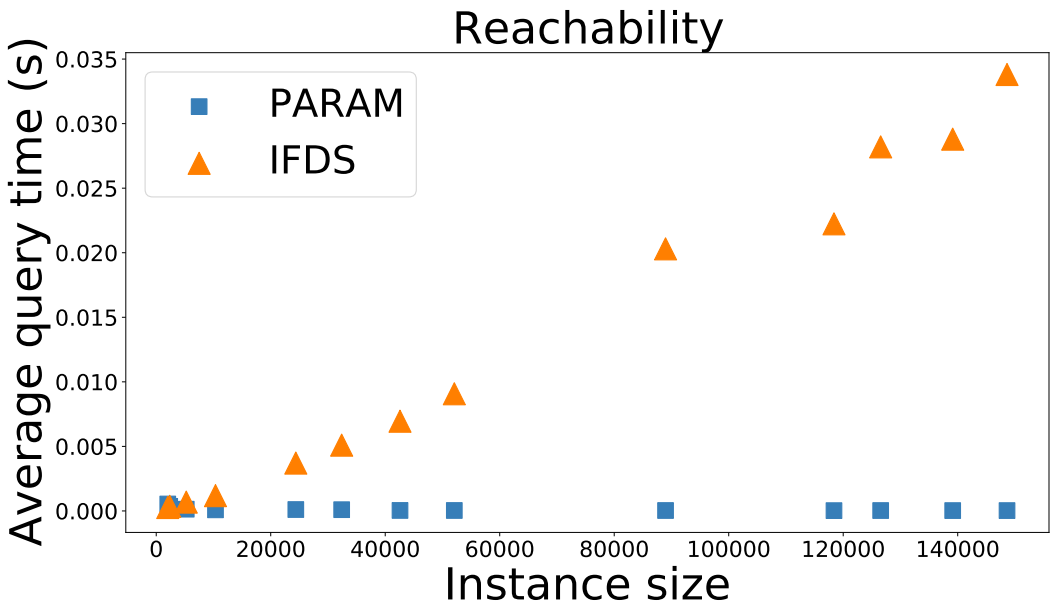
Query:

- Average: 0.11ms.
- Maximum: 0.53ms.

`IFDS`'s query:

- Average: 12.3ms.
- Maximum: 33.80ms.

`IFDS/PARAM`: 390.55.



Experiments: reachability analysis

Preprocessing:

- Average: 0.93s.
- Maximum: 1.53s.

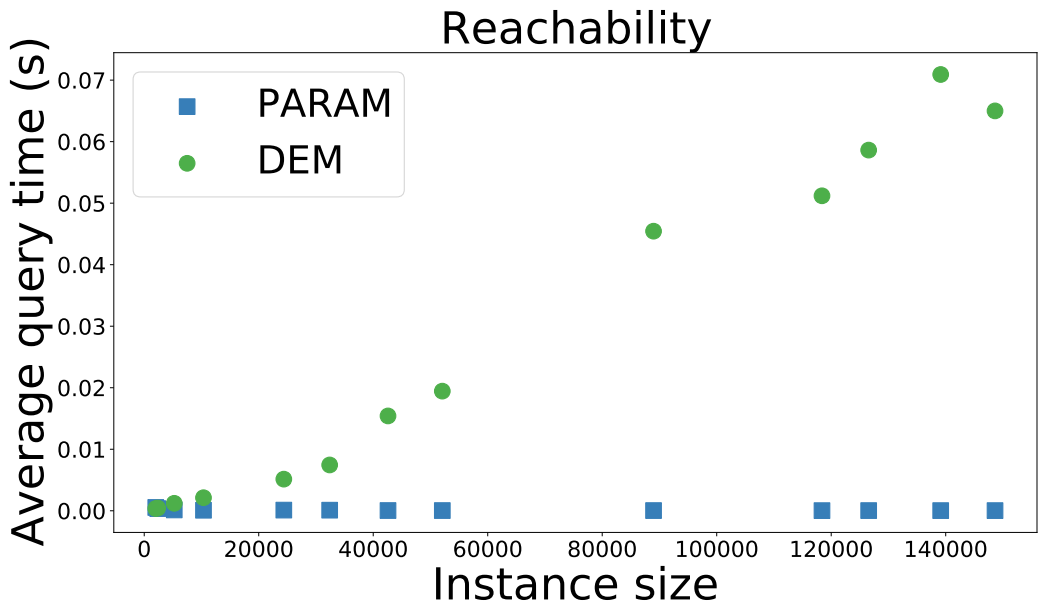
Query:

- Average: 0.11ms.
- Maximum: 0.53ms.

`DEM`'s query:

- Average: 26.36ms.
- Maximum: 70.91ms.

`DEM/PARAM`: 848.13.



Experiments: null-pointer analysis

Preprocessing:

- Average: 41.80s.
- Maximum: 140.85s.

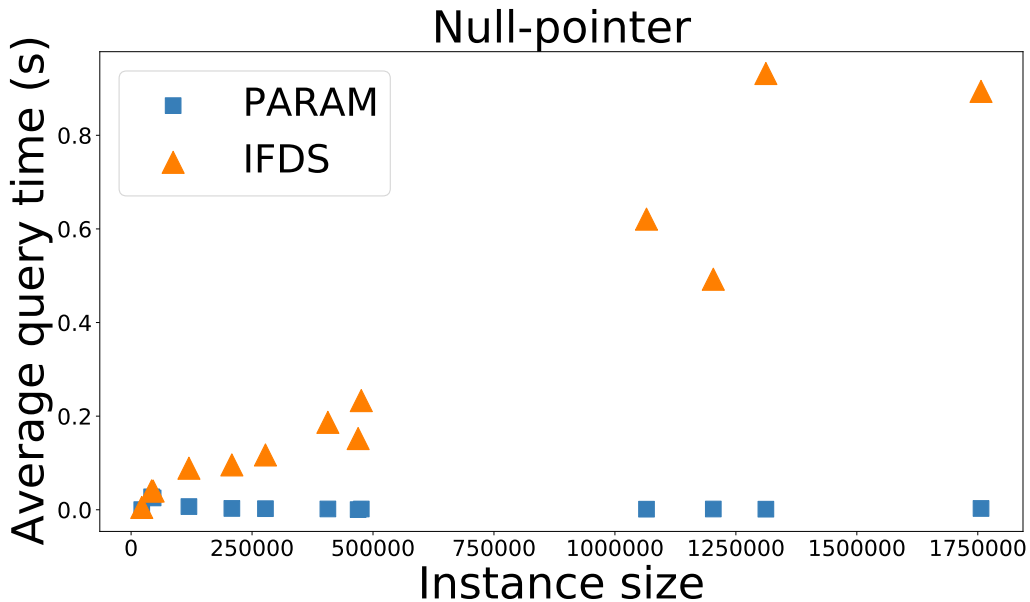
Query:

- Average: 5.84ms.
- Maximum: 27.63ms.

`IFDS`'s query:

- Average: 299.91ms.
- Maximum: 932.04ms.

`IFDS/PARAM`: 202.92.



Experiments: null-pointer analysis

Preprocessing:

- Average: 41.80s.
- Maximum: 140.85s.

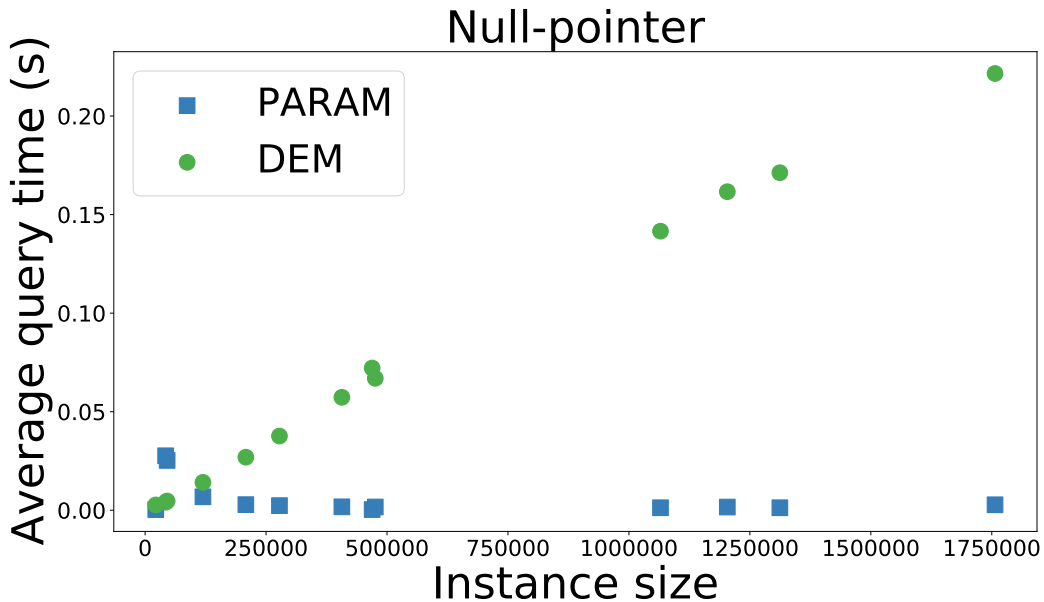
Query:

- Average: 5.84ms.
- Maximum: 27.63ms.

`DEM`'s query:

- Average: 75.58ms.
- Maximum: 221.58ms.

`DEM/PARAM`: 56.86.



Experiments: possibly-uninitialized variables analysis

Preprocessing:

- Average: 89.44s.
- Maximum: 265.31.

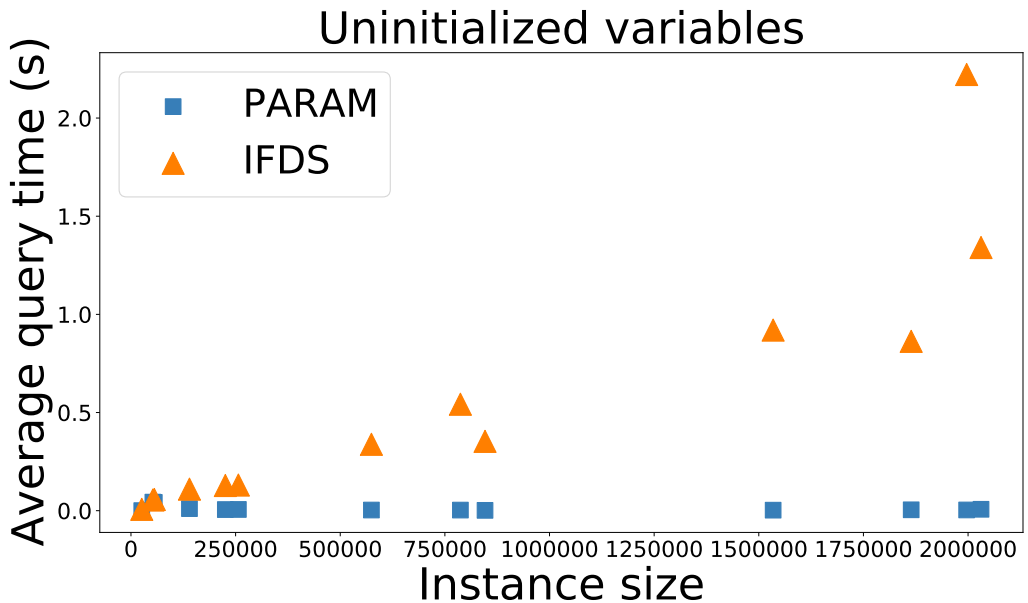
Query:

- Average: 10.39ms.
- Maximum: 43.70ms.

`IFDS`'s query:

- Average: 543.53ms.
- Maximum: 2221.90ms.

`IFDS/PARAM`: 143.96



Experiments: possibly-uninitialized variables analysis

Preprocessing:

- Average: 89.44s.
- Maximum: 265.31.

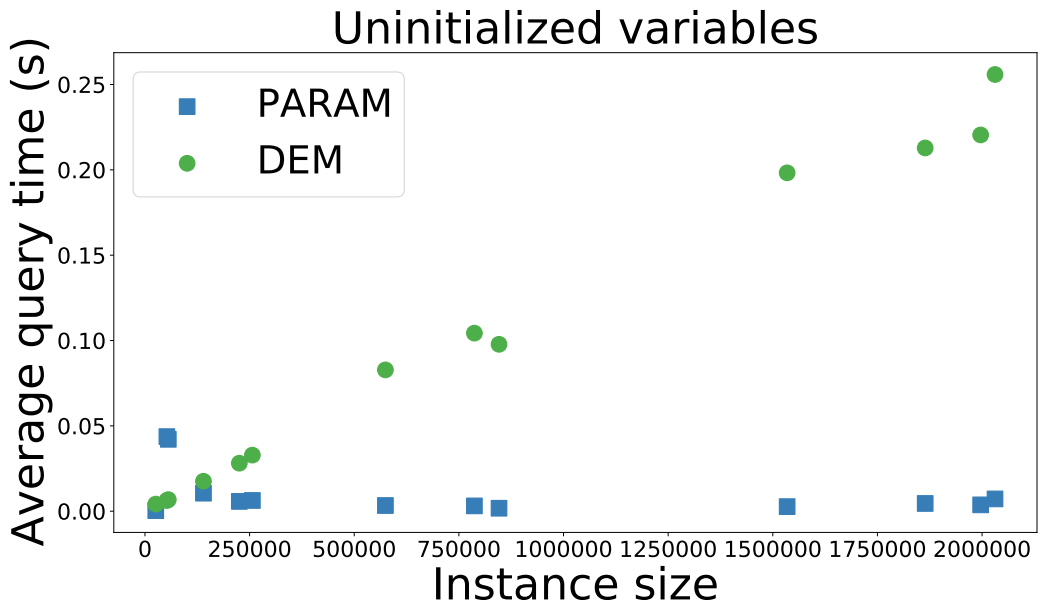
Query:

- Average: 10.39ms.
- Maximum: 43.70ms.

`DEM`'s query:

- Average: 97.54ms.
- Maximum: 255.86ms

`DEM/PARAM`: 27.25.



Conclusion

- Identify and exploit a new sparsity parameter: treedepth of call graphs.
- Fast parameterized algorithm for general on-demand IFDS.
- Theoretical improvement over previous works.
- Experimentally outperforming the standard IFDS algorithms by two orders of magnitude.

Approach	General?	Preprocessing	Query
Reps et. al. (POPL'95)	✓	$O\left(n \cdot D^3\right)$	
Horwitz et. al. (FSE'95)	✓	$O\left(n \cdot D^3\right)$	
Chatterjee et. al. (ESOP'20)	✗	$O\left(n \cdot D^3\right)$	$O\left(\lceil D / \lg n \rceil\right)$
Our result	✓	$O\left(n \cdot D^3 \cdot \mathtt{td}\right)$	$O\left(D^3 \cdot \mathtt{td}\right)$

Publications

— A.K. Goharshady, A.K. Zaher, *"Efficient Interprocedural Data-Flow Analysis using Treedepth and Treewidth,"* in VMCAI'23.

— G.K. Conrado, A.K. Goharshady, K. Kochekov, Y.C. Tsai, A.K. Zaher, *"Exploiting the Sparseness of Control-flow and Call Graphs for Efficient and On-demand Algebraic Program Analysis,"* in OOPSLA'23.

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[2] S. Horwitz, T. W. Reps, and S. Sagiv, "Demand interprocedural dataflow analysis," in FSE, 1995, pp. 104–115.

[3] K. Chatterjee, A. K. Goharshady, R. Ibsen-Jensen, and A. Pavlogiannis, "Optimal and perfectly parallel algorithms for on-demand data-flow analysis," in ESOP, 2020, pp. 112–140.

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[6] W. Nadara, M. Pilipczuk, and M. Smulewicz, "Computing treedepth in polynomial space and linear FPT time," CoRR, vol. abs/2205.02656, 2022.

[7] H. Dell, C. Komusiewicz, N. Talmon, and M. Weller, "The PACE 2017 Parameterized Algorithms and Computational Experiments Challenge: The Second Iteration," in IPEC, 2018, pp. 30:1–30:12.

[8] Łukasz Kowalik, M. Mucha, W. Nadara, M. Pilipczuk, M. Sorge, and P. Wygocki, "The PACE 2020 Parameterized Algorithms and Computational Experiments Challenge: Treedepth," in IPEC, 2020, pp. 37:1–37:18.