Exploiting the Sparseness of Control-flow and Call Graphs for Efficient and On-demand Algebraic Program Analysis

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香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Agenda

1. Context and contribution
2. Algorithms
3. Experiments and conclusion

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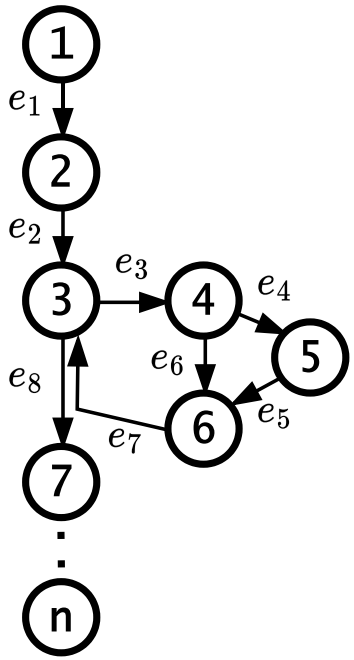

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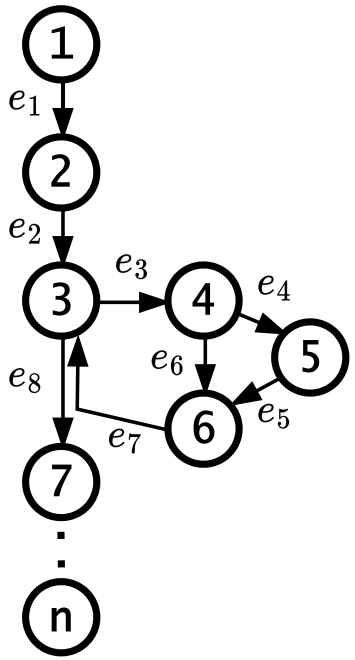

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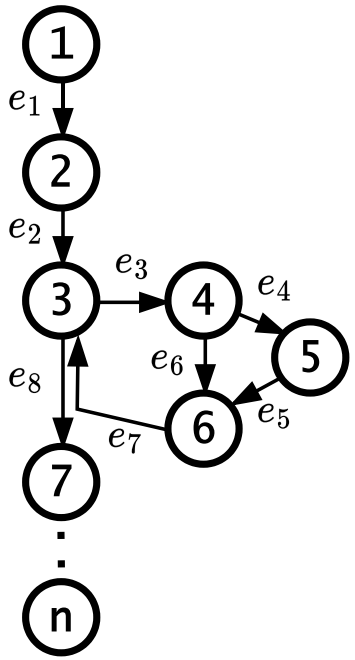

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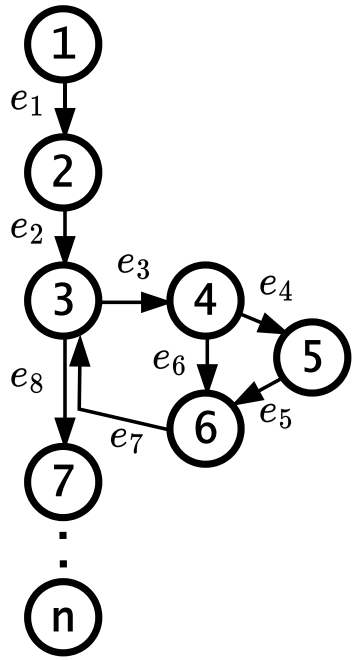

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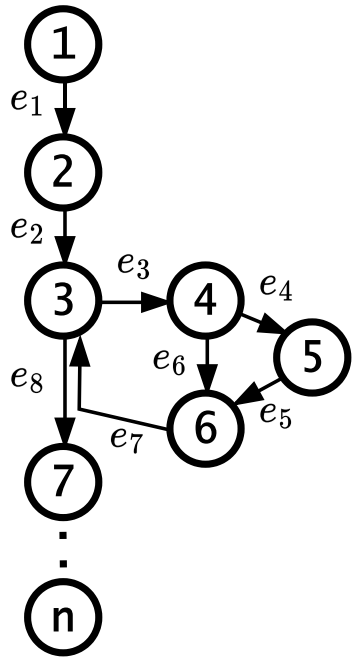

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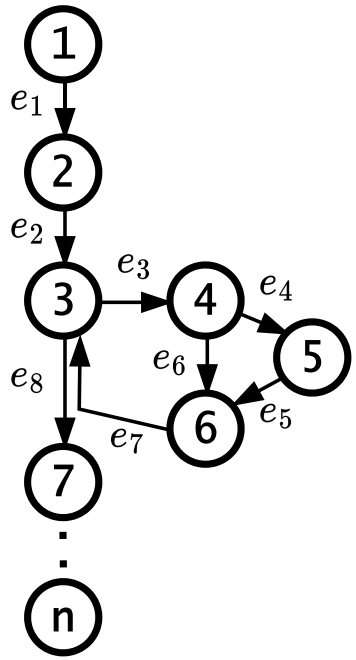

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Applications: numerical invariant generation, predicate abstraction.

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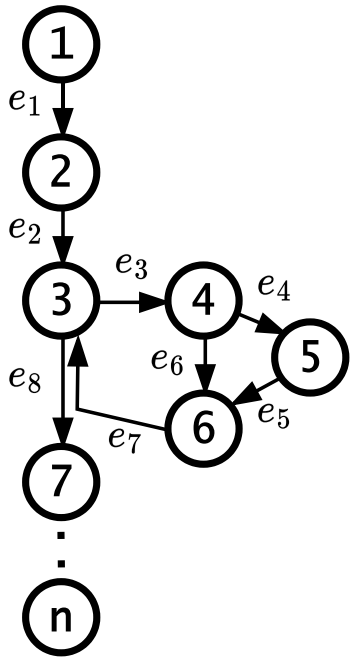

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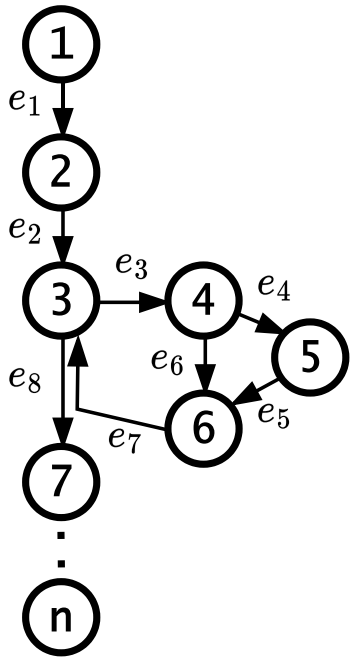

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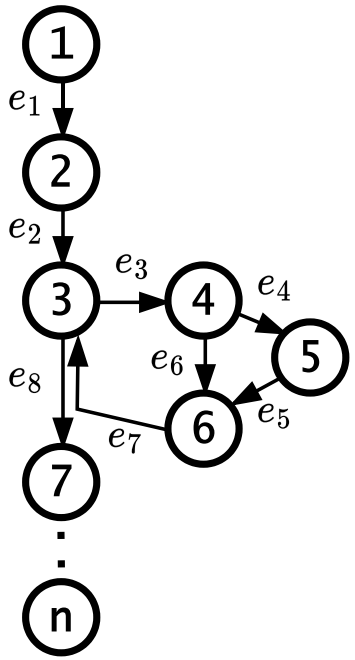

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- Query: express input queries as combination of precomputed queries.
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Intra-procedural case: we **exploit sparsity of control-flow graphs.**

Inter-procedural case:
- We assume function summaries are computed and are given in input.
- We additionally **exploit sparsity of call graphs.**

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- $n = |V| \approx |E|$, cost per algebra operation is k:
- *Tarjan's algorithm:*
  - Works on reducible flow graphs ($\approx$ CFGs).
  - Answers all queries $(i, -)$ for a fixed i in $O(n\alpha(n) \cdot k)$.
  - Doesn't suit our on-demand setting:
    - for n queries with different i's, naive repetition $\implies \Omega(n^2)$ time.

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Algorithm #2: exploits treewidth.

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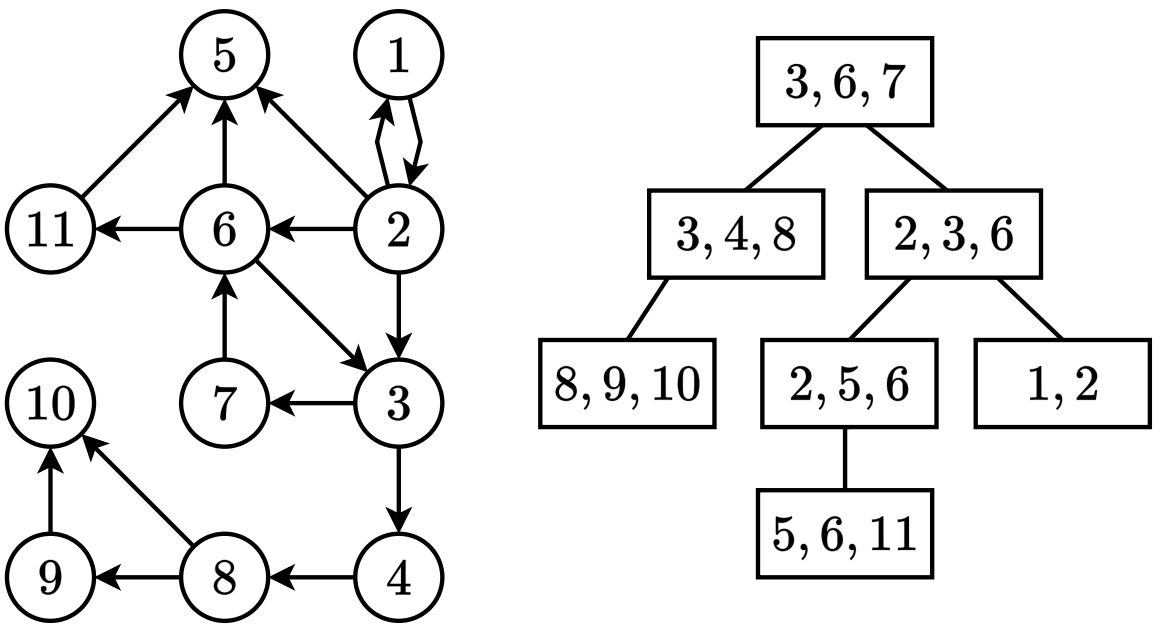

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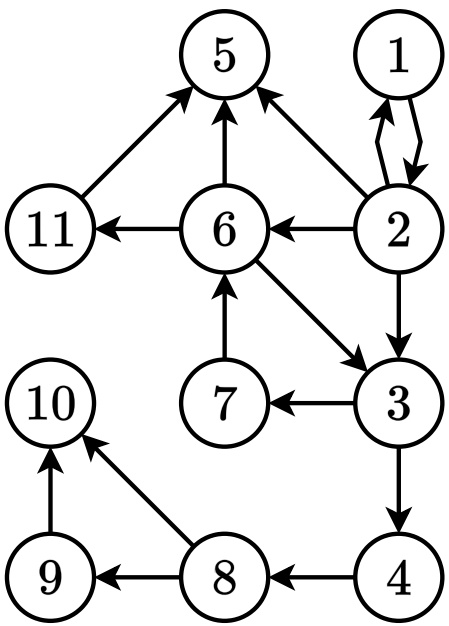
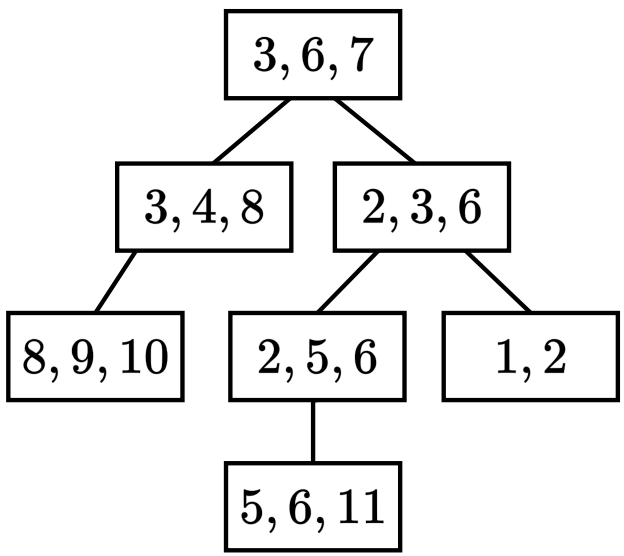
- CFGs have constant treewidth (Thorup '98).





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$\Longrightarrow$ takes too much time and space.

Better: precompute only *"special queries"* having a certain form s.t.,

- **Expressiveness:** a general query can be expressed with special queries.
- **Space:** the number of special queries should be $\ll n^2$.
- **Time:** total runtime should be small.

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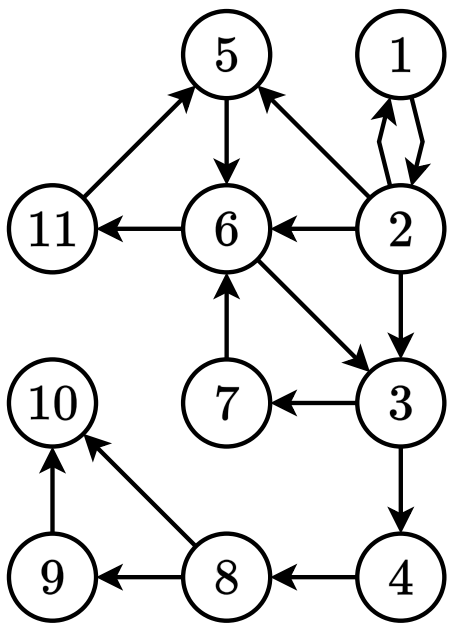

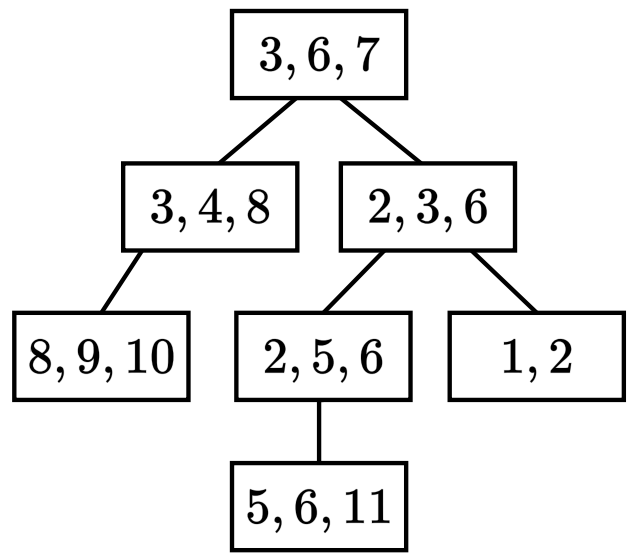

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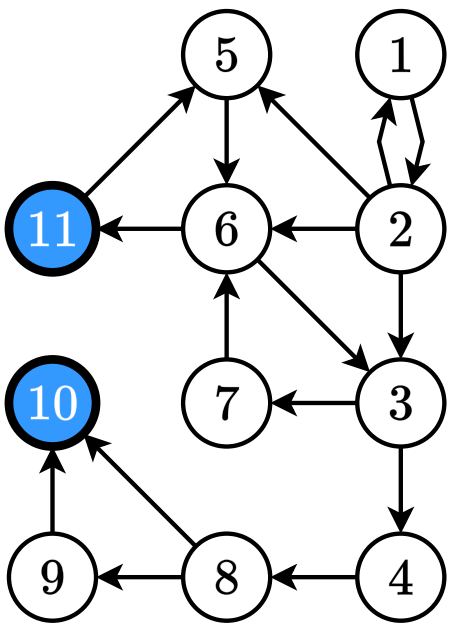

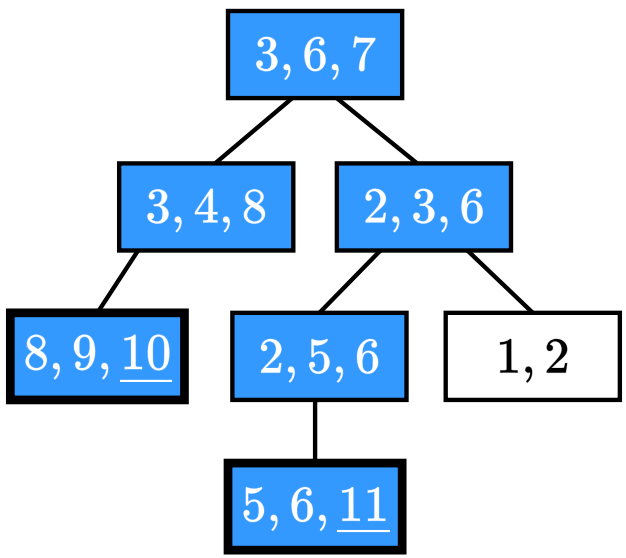

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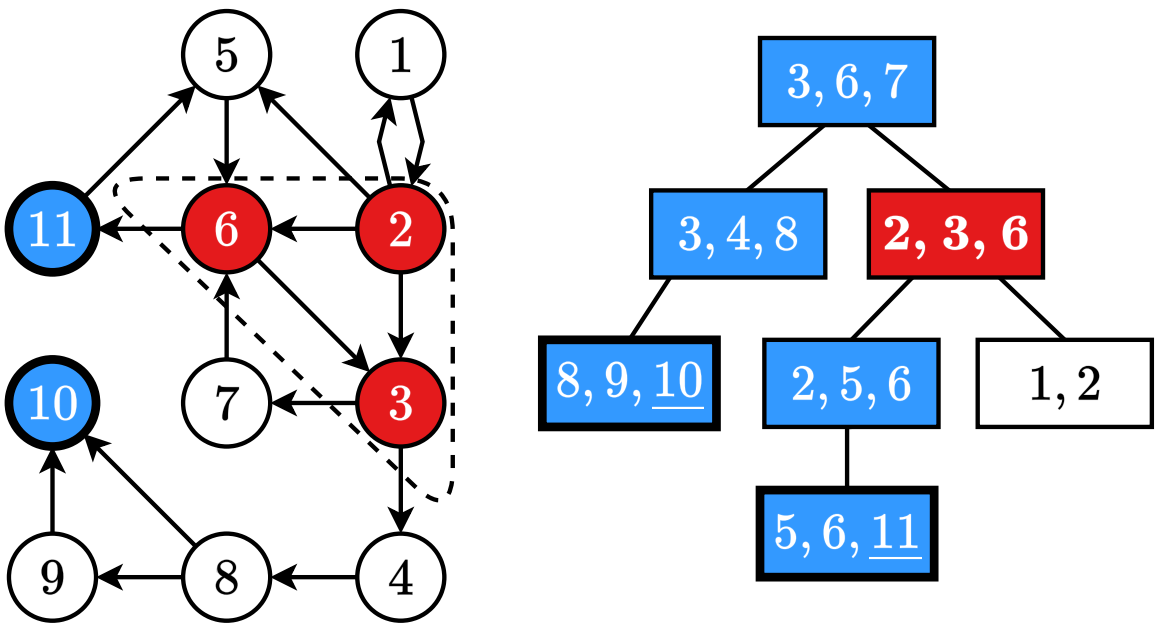

Intra-procedural algorithm #2

We look at the tree decomposition T of the CFG G.

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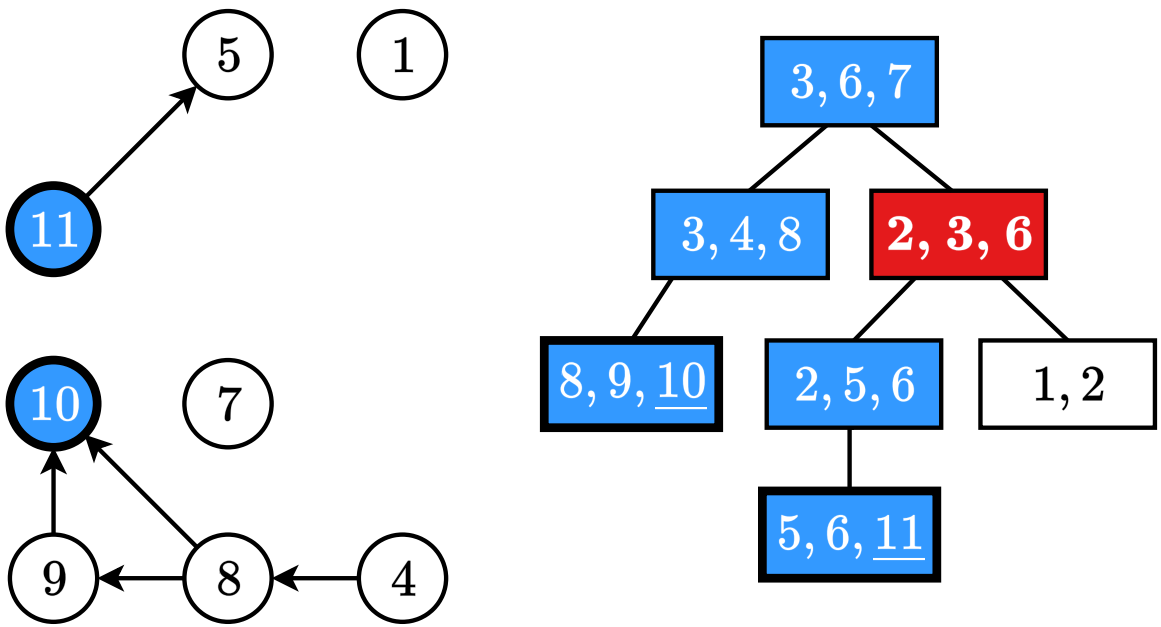

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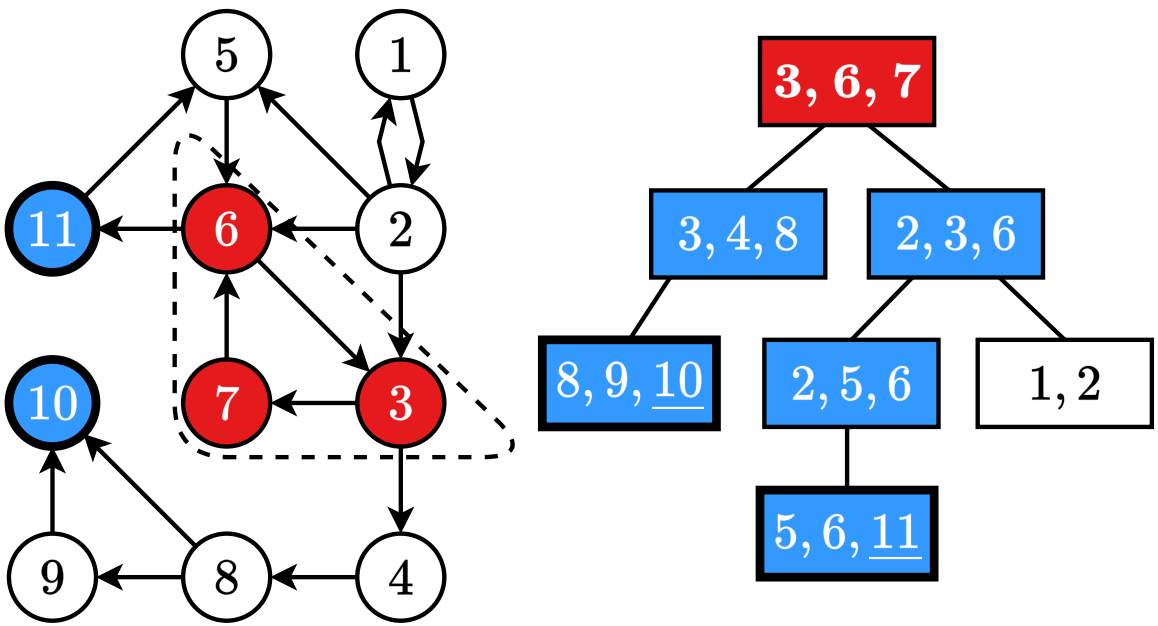

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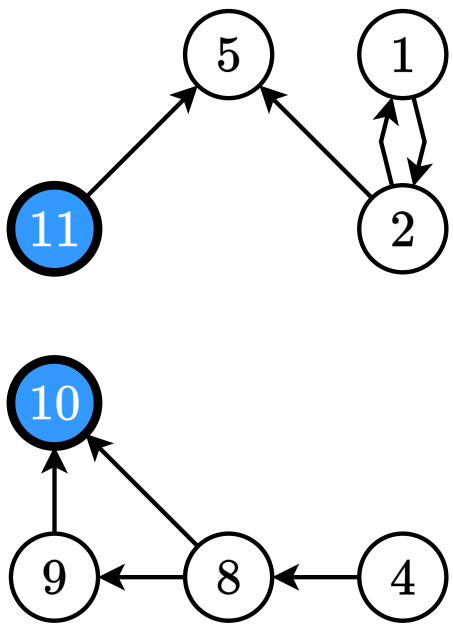

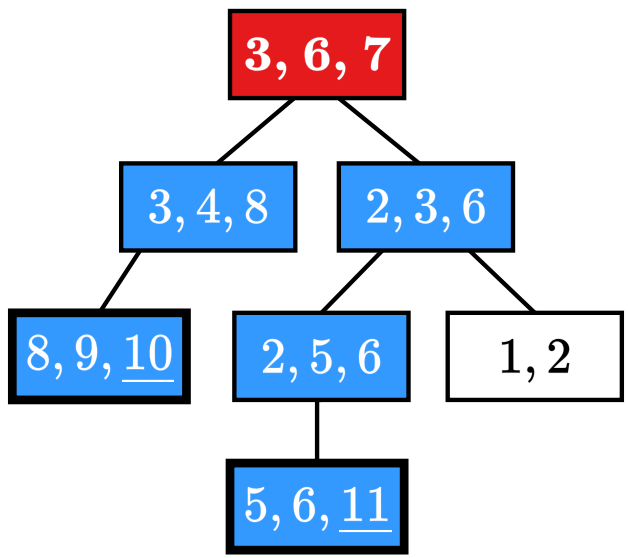

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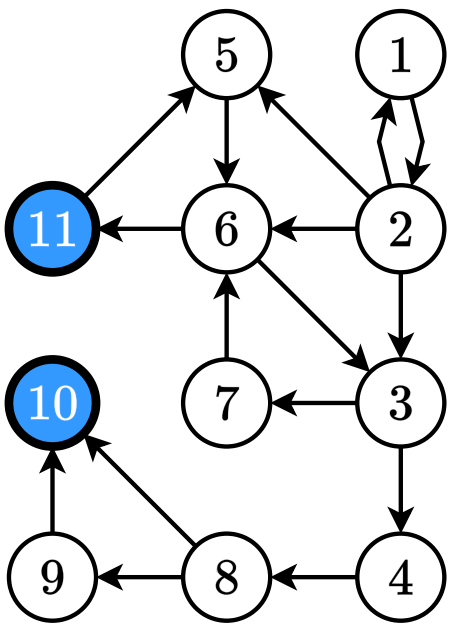
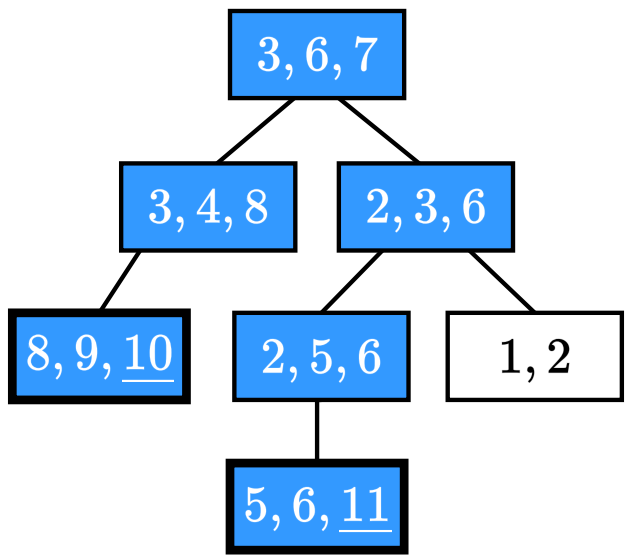
A way to break the query!

For <u>any</u> bag $b \in P_{b_u,b_v}$, $[\![\rho_{u,v}]\!] = \bigoplus_{w \in V_b} [\![\rho_{u,w}]\!] \otimes [\![\rho_{w,v}]\!]$





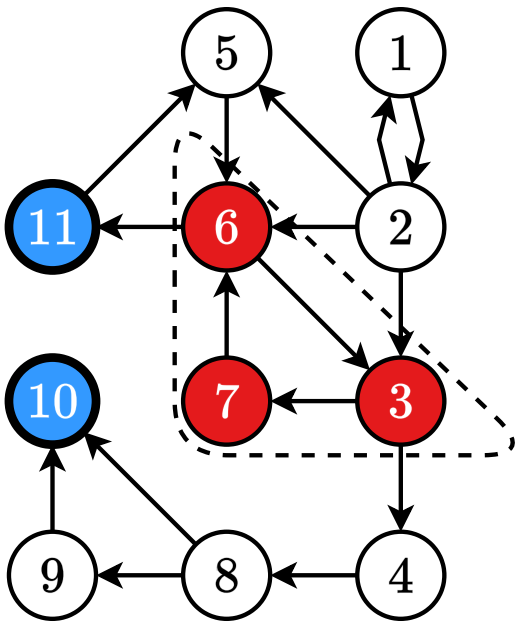
Intra-procedural algorithm #2 via tree decomposition

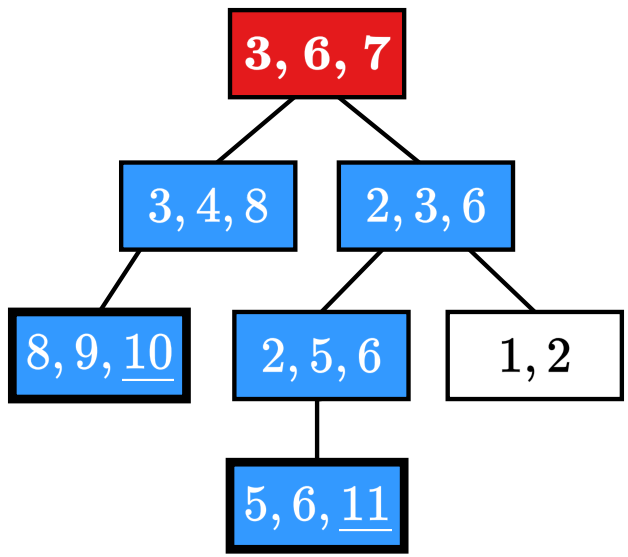
Idea:

- Always choose b to be the least common ancestor bag of b_u, b_v in T.

Picking $V_b = \{2, 3, 6\}$,

$$[\![\rho_{11,10}]\!] = ([\![\rho_{11,2}]\!] \otimes [\![\rho_{2,10}]\!]) \oplus ([\![\rho_{11,3}]\!] \otimes [\![\rho_{3,10}]\!]) \oplus ([\![\rho_{11,6}]\!] \otimes [\![\rho_{6,10}]\!])$$





Intra-procedural algorithm #2 via tree decomposition

Idea:

- Always choose b to be the least common ancestor bag of b_u, b_v in T.
- Define "special queries" to be all pairs (u, v) where b_u is an ancestor/descendant of a b_v.

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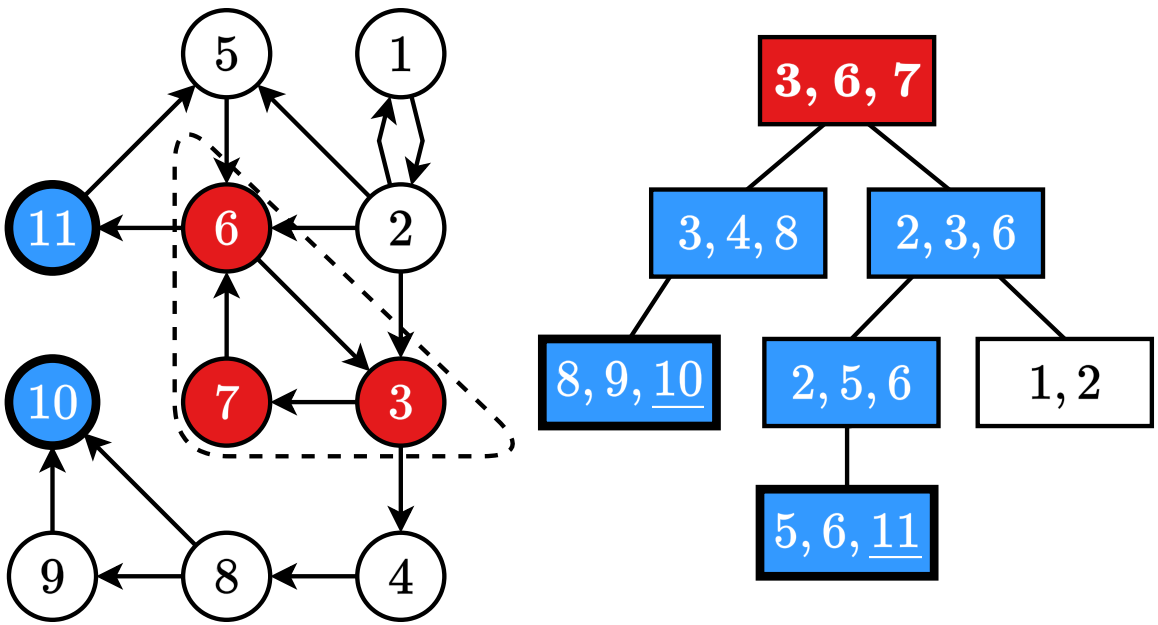

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Preprocessing: precompute all special queries.
Query (u, v):

- Find b_u and b_v.
- Let $b_{LCA} := LCA(b_u, b_v)$, the least common ancestor in T.
- return

$$[\![\rho_{u,v}]\!] = \bigoplus_{w \in b_{LCA}} [\![\rho_{u,w}]\!] \otimes [\![\rho_{w,v}]\!].$$

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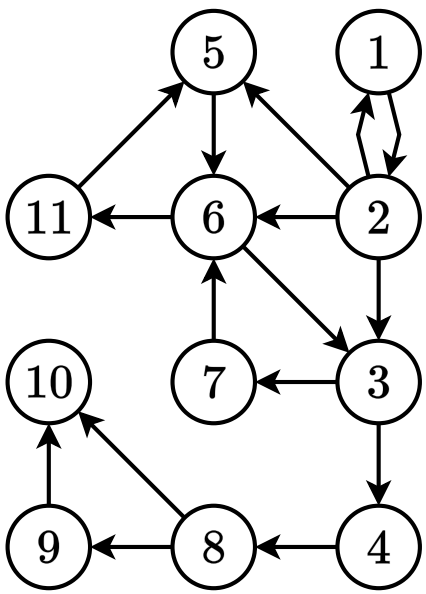
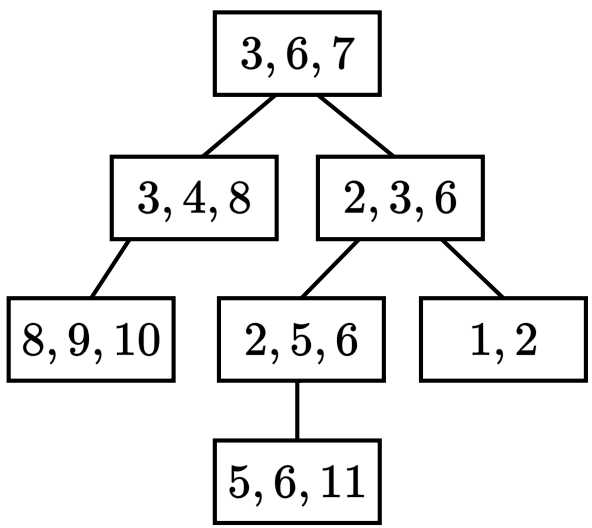
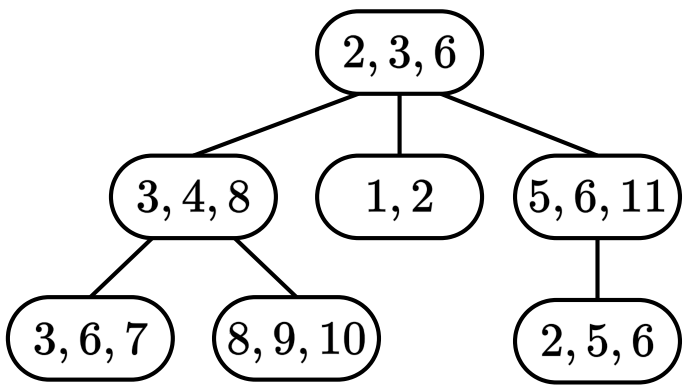
Number of "special queries" $= O(n \cdot$ height of the TD$)$.
Problem: if tree decomposition is too long $\implies O(n^2)$ special queries.

Intra-procedural algorithm #2 via tree + centroid decomp.

Solution: build a *centroid decomposition T' of the tree decomposition T, which has height $O(\log n)$*.



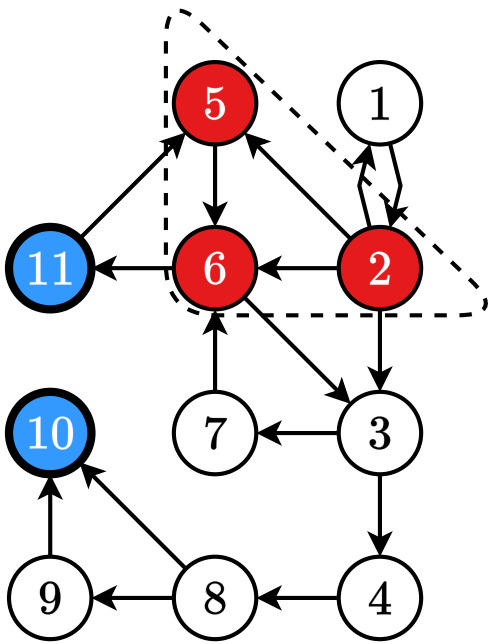


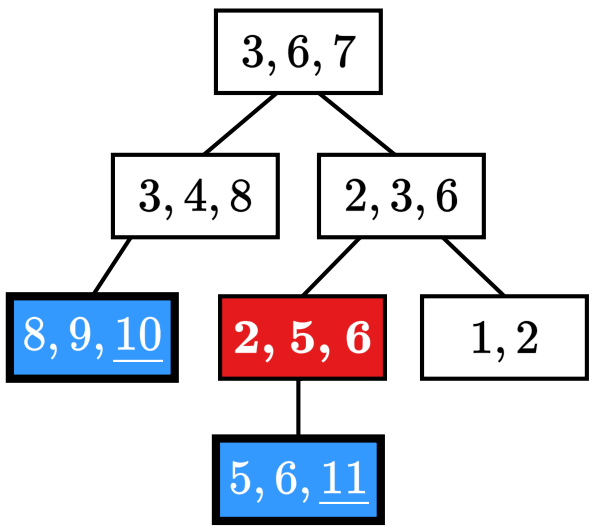


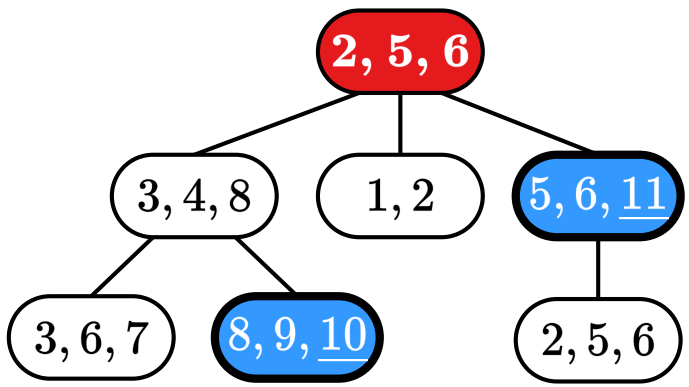
Intra-procedural algorithm #2 via tree + centroid decomp.

Solution: build a *centroid decomposition T' of the tree decomposition T, which has height $O(\log n)$.*

- Apply previous slide with b_{LCA} being LCA in T' not T.







Intra-procedural algorithm #2 via tree + centroid decomp.

Solution: build a *centroid decomposition* T' *of the tree decomposition* T, *which has height* $O(\log n)$.

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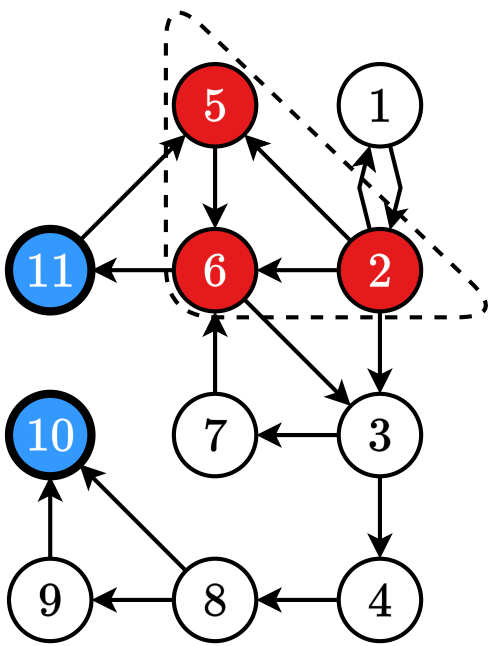
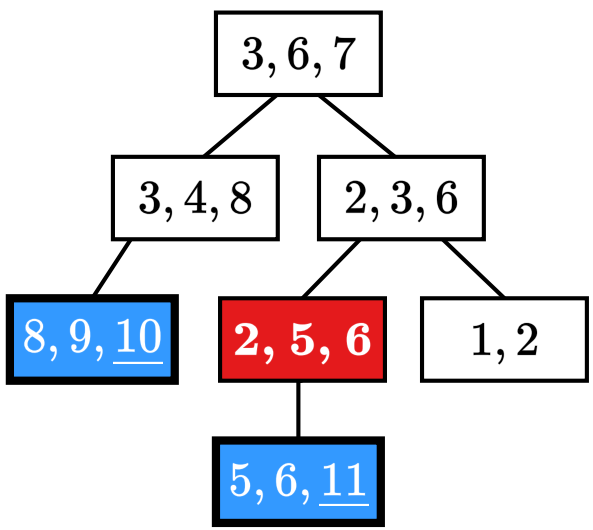
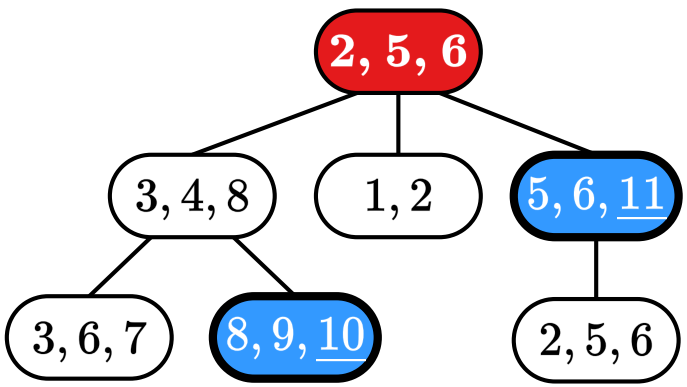
Number of "special queries" $= O(n \cdot \log n)$.
We precompute all special queries in $O(n \cdot \log n \cdot k)$.







Inter-procedural algorithm

– *We extend our algorithm by exploiting sparsity of the call graph.*
– *Capture the sparsity using treedepth.*

See the paper for details.

Agenda

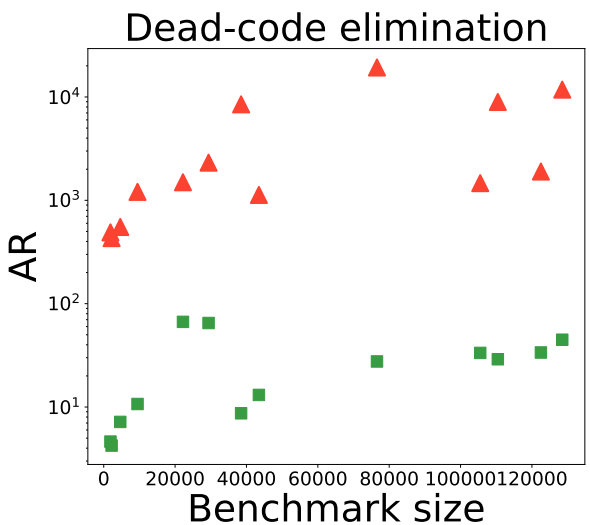
1. Context and contribution
2. Algorithms
3. Experiments and conclusion

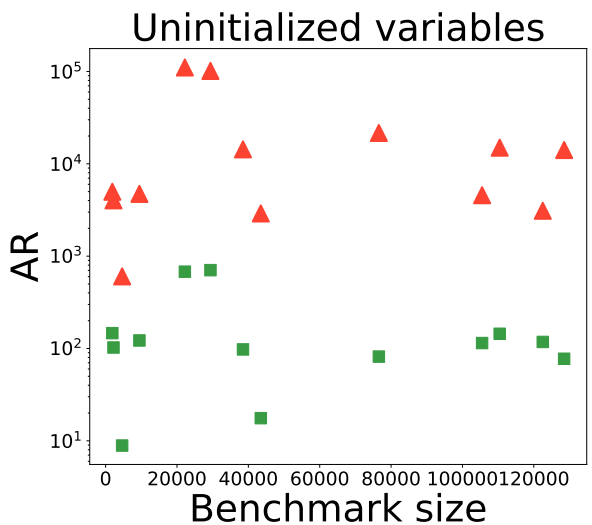
Experiments

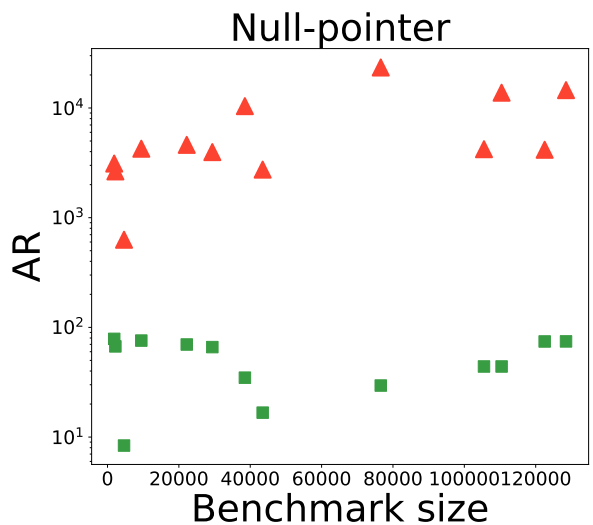
1. IFDS dataflow analyses (reachability, uninitialized variables, null-ptr):

- Each algebra element is the graph representation of IFDS.
- Used programs from DaCapo benchmarks.

Comparison of our algorithms vs. running Tarjan's algo. at every query:







Experiments

2. Analysis of boolean programs:

- Each algebra element is a state transformer represented by a BDD.
- Used boolean programs generated from applying Predicate Abstraction on Windows drivers.

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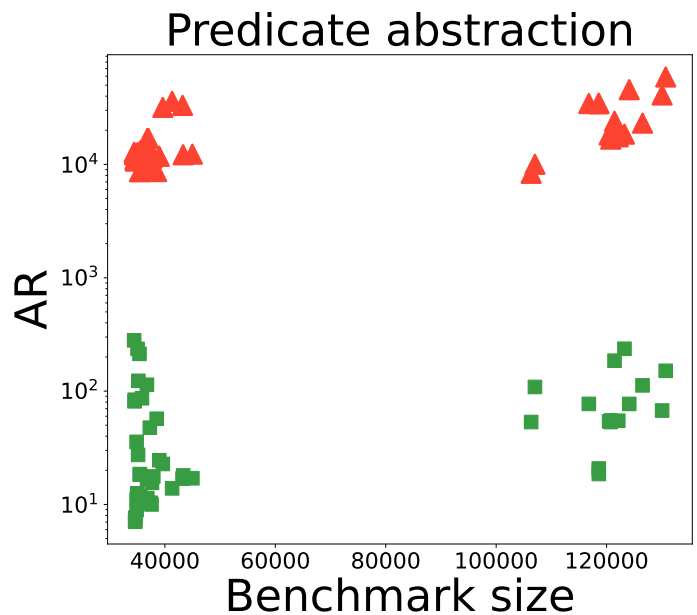

Conclusion

Fast algorithms for on-demand algebraic program analysis.

- Exploiting sparseness of CFGs (via treewidth) to handle the intra-procedural queries.
- Exploiting sparseness of CGs (via treedepth) to extend the solution to the inter-procedural case.
- Experiments showing efficiency in comparison with using Tarjan's algorithm.

Intra-procedural algorithms

Intra-procedural on-demand algebraic program analysis

Offline input: (can be preprocessed)

- *a CFG $G = (V, E)$ of a single function,*
- an algebra $(A, \oplus, \otimes, \circledast, \overline{0}, \overline{1})$,
- a semantic function $[\![\cdot]\!] : E \to A$.

Online input: a series of queries (i, j), each is a pair of program points.

Output: for each query (i, j), compute $[\![\rho_{i,j}]\!]$ where $\langle \rho_{i,j} \rangle = \mathit{Paths}_G(i, j)$.

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—*Kleene's NFA-to-regexp translation:*
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—*Tarjan's algorithm:*
- Works on reducible flow graphs ($\approx$ CFGs).
- Answers all queries $(i, -)$ for a fixed i in $O(n\alpha(n) \cdot k)$.
- Doesn't suit our on-demand setting:
  - for n queries with different i's, naive repetition $\implies \Omega(n^2)$ time.

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Algorithm #2:

- Operates on the "tree decomposition" of the CFG.
- Assumes CFGs have constant treewidth: more robust assumption.
- Preprocessing: $O(n \cdot \log n \cdot k)$; query $O(k)$.

Intra-procedural algorithm #1

$$P := \sigma \mid P; P \mid \texttt{branch}_l\ P, P, \ldots, P\ \texttt{end}_l \mid \texttt{loop}_l\ P\ \texttt{end}_l \mid \texttt{break}_l \mid \texttt{continue}_l$$

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Preprocessing:

- Structurally recursive: before processing P, process its subprograms first.

P

$\sigma_1;$

$\sigma_2;$

$\texttt{branch}_3$

$P_1,$

P_2

$\texttt{end}_3;$

$\sigma_4;$

$\texttt{loop}_5$

P'

$\texttt{end}_5;$

σ_6

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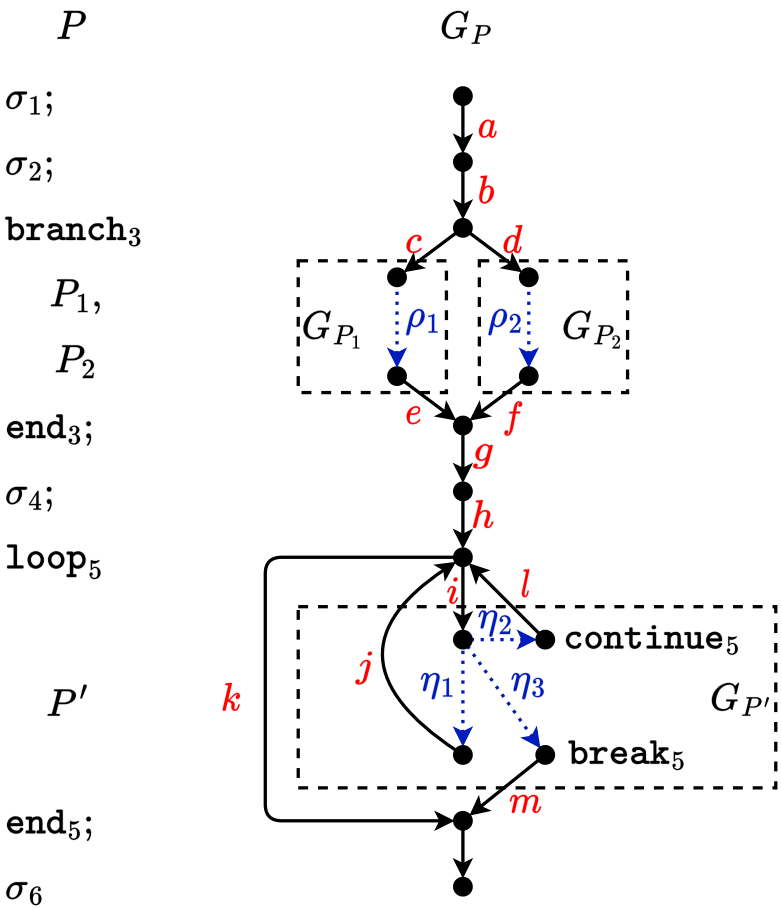

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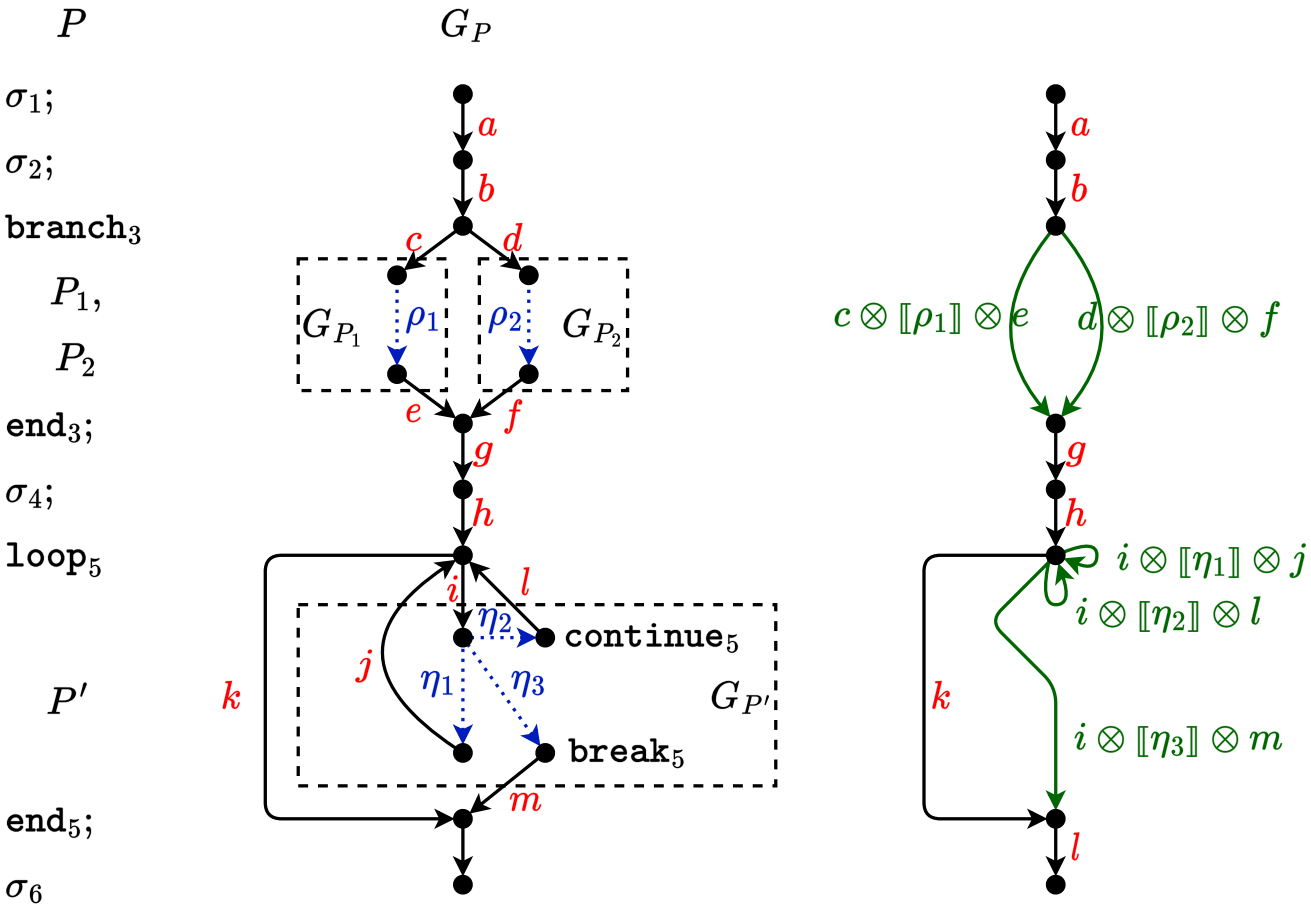

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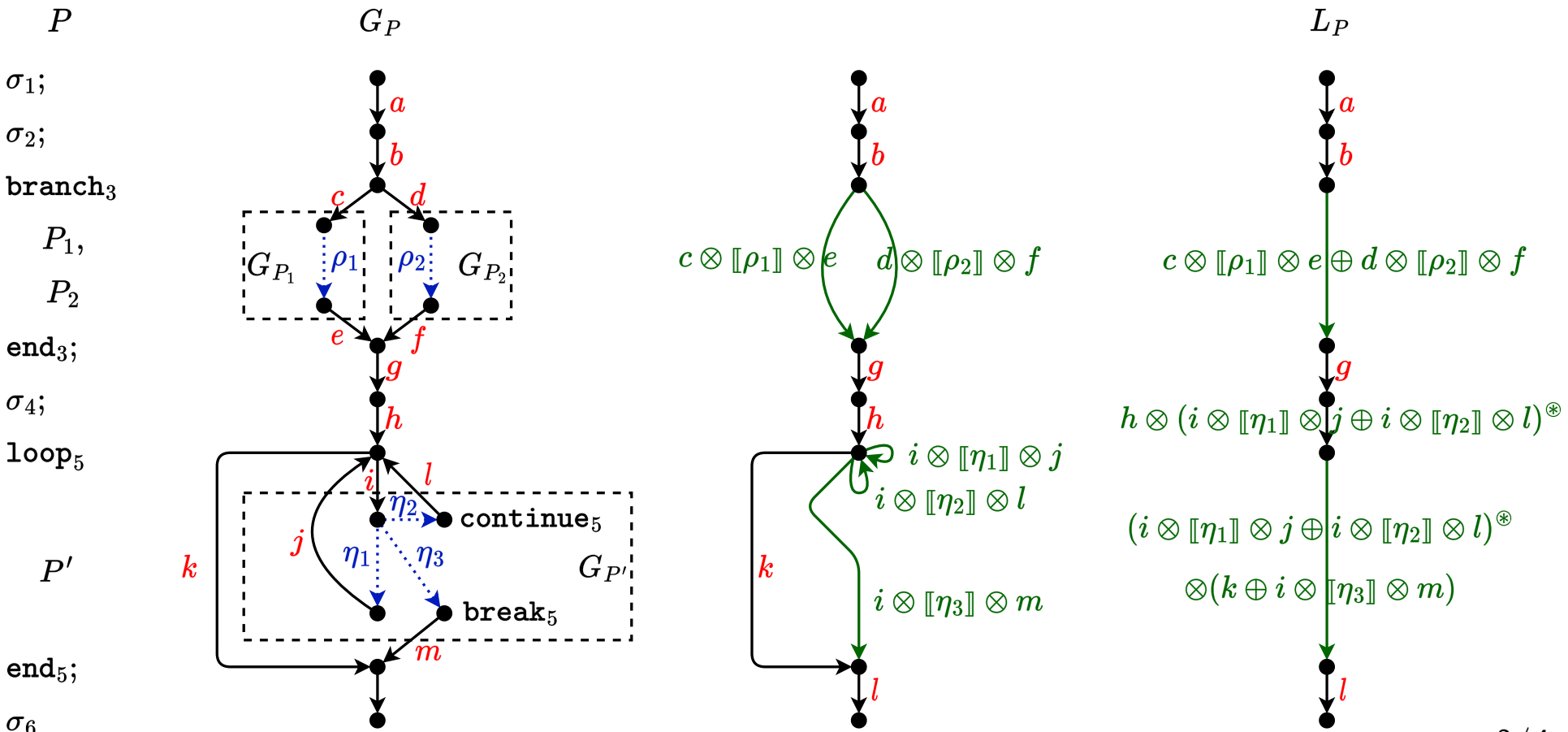

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To answer a query (i, j), a path $i \rightsquigarrow j$ either:

- Visits some node at the top-level,
  — can be answered with the sqrt-tree.
- Or it doesn't
  — reduces to a query in a subprogram with smaller depth.

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- Structurally recursive: before processing P, process its subprograms first.
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To answer a query (i, j), a path $i \rightsquigarrow j$ either:

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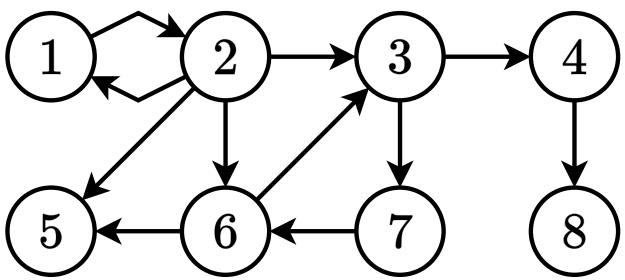

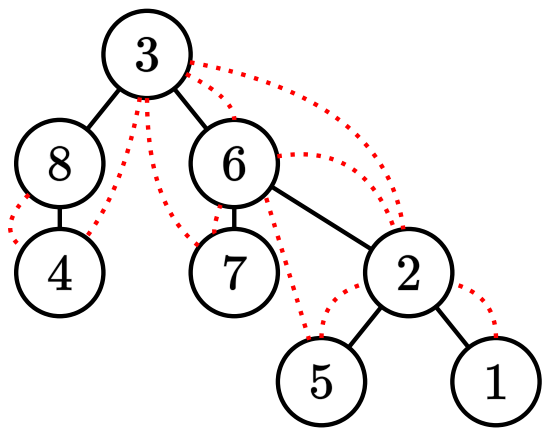

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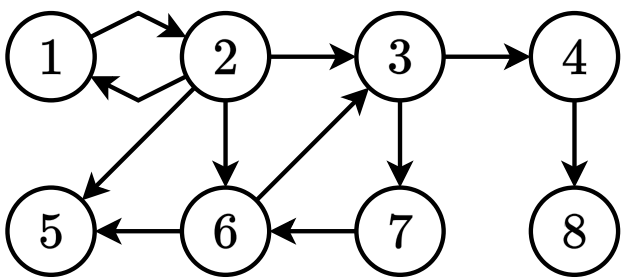

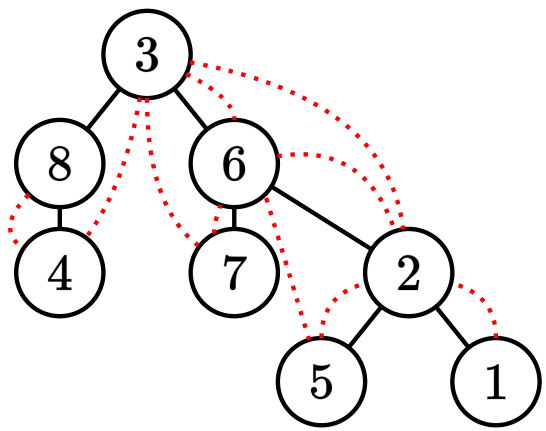

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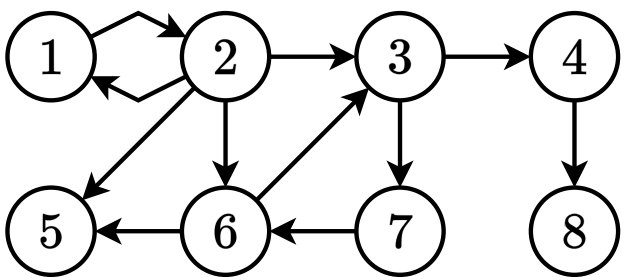
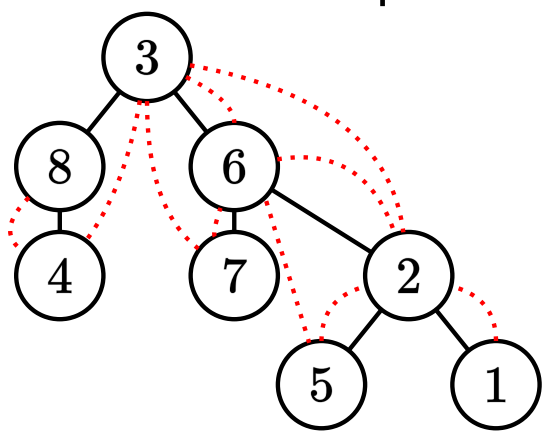
- We exploit this assumption to efficiently solve the inter-procedural case.





Inter-procedural algorithm

Inter-procedural on-demand algebraic program analysis

Offline input: (can be preprocessed)

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Answering call-graph queries: find depth decomp. $\to$ convert to tree decomp $\to$ apply treewidth-based algorithm. Done!